where $<e_0>$ is the mean kinetic energy at the 0 K and $\varepsilon_F$ is the Fermi energy of the nucleons at 0 K.

Show that the total excitation energy of the nucleus at the temperature $T$ is equal to $11 (kT)^2$ MeV.

5. Using Eq. (5.7-9) for the $\beta^+$ disintegration energies of the mirror nuclei, draw the graph of $Q_{\beta^+}$ against $A^{2/3}$ for the five mirror nuclei with $A = 7, 9, 11, 13$ and $15$. Find the value of $a$ from this graph. Find out the disintegration energies from literature. Assuming the expectation value of the potential energy of a harmonic oscillator to be equal to half the total energy, use Eqs. (9.10-5a) and (9.10-5b) to show that the mean squared radius is given by $<r^2> = (\hbar/2m\omega) N^2$ for large $N$; $N$ is the number of nucleons up to a given maximum value of $\lambda$.

Hence show that the level spacing is given by $\hbar\omega = 40 A^{-1/3}$ Mev (Eq. 9.10-5b).

6. The electromagnetic spin orbit energy is given by

$$\varepsilon_{s.o.} = \frac{Ze^2 \hbar^2}{8 \pi \varepsilon_0 m c^2} \frac{1}{<r^2>} (I \cdot s)$$

Use this expression to obtain the spin-orbit splitting between the two states with $j = l + 1/2$ and $j = l - 1/2$ in the case of a nucleus with $Z = 14$ and $A = 28$. Assume the nuclear radius to be given by $R = <r> = 1.2 \times A^{1/3}$ fm. Compare this with the splitting given by the empirical formula (9.10-14a).

10

Nuclear Reactions (I)

10.1 Discovery of artificial transmutation of elements

Rutherford’s experiment

From the beginning of civilization, people in different parts of the world had an intense desire to know whether baser metals like iron, copper etc. could be transformed into the noble metals like gold or silver. There were many who thought that such transformation was possible. In the middle ages, a pseudo-science known as alchemy, had flourished in Europe. Alchemists claimed that they could transform baser metals into noble metals, though there was little scientific basis for their claim. In fact many of them had to pay dearly for their fraudulent activities.

The discovery of radioactivity at the beginning of the present century led to the realisation that the radioactive elements spontaneously transformed into other elements. Following this discovery, the ancient dream of the alchemists was again revived in the minds of scientists regarding the possibility of transforming one element into another.

From our knowledge about the structure of the atomic nuclei it is clear that if we can change the number of protons or neutrons or both inside the nucleus, then it would be possible to bring about a transformation of the nucleus. If the proton number $Z$ is changed, then it is possible to transform one element into another. On the other hand, if the neutron number $N$ is changed, then one isotope of an element will be transformed into another isotope of the same element.

The main difficulty in producing the transformation of a nucleus artificially is the very tight binding of the nucleons inside the nucleus. To remove a nucleon from a nucleus, we must supply it a quantity of energy at least equal to the energy of its binding within the nucleus, which is usually of the order of a few MeV. This energy can be supplied by introducing a nuclear particle (e.g., a proton, neutron, deuteron or an $\alpha$-particle) into the nucleus from outside. Except neutrons, all the others are positively charged and hence are strongly repelled by the positive charge of the nucleus. So they must be highly energetic to be able to enter the nucleus to bring about a nuclear transformation.
Lord Rutherford was the first to produce artificial transformation (transmutation) of a nucleus in 1919, using the highly energetic $\alpha$-particles from naturally radioactive substances like radium as projectiles.\footnote{It may be mentioned that as early as 1916, the Indian physicist D.M. Bose working in the laboratory of Regener in Germany found in a cloud chamber photograph, the evidence for the emission of a charged particle from the end of an $\alpha$-track with a range much longer than the range of the $\alpha$-particle in the gas filling the chamber. Another shorter track which was much thicker also came out from the same point. It was clearly the case of an $\alpha$-induced nuclear transformation which, however, could not be recognised as such by Bose.}

The apparatus used by Rutherford is shown in Fig. 10.1.

![Fig. 10.1. Rutherford's apparatus for producing artificial disintegration of nuclei.](image)

An air-tight glass chamber A which could be evacuated with the help of a vacuum pump and then filled with any desired gas, contained a small sample D of a naturally radioactive substance.

$\alpha$-particles from the source D travelled through the gas in the chamber towards a thin window covering a port at the other end on the chamber wall. Outside the window, there was a fluorescent screen F on which scintillations were produced by the energetic charged particles falling on it. Thin metallic absorber foils S could be interposed between the window and F. The scintillations could be observed with the help of a microscope M.

The distance from D to the window was kept greater than the range of the $\alpha$-particles from the source in the gas within the chamber. No scintillation could be observed when the chamber was filled with CO$_2$ or oxygen. However, when the chamber was filled with dry air or nitrogen, scintillations could be observed, even when the distance between the source and the screen F was 40 cm or more air-equivalent.

Rutherford identified the particles producing the scintillations as protons by deflecting them by a magnetic field. Their much longer range compared to that expected for the elastically scattered protons from hydrogen gas (28 cm) excluded the possibility of their origin from any hydrogen gas which might be mixed with nitrogen as impurity.

Rutherford explained his observations in the following way. When the very high velocity $\alpha$-particles made head-on collisions with the nitrogen nuclei $^{14}\text{N}$ some of them were captured by the latter. The composite system, which was formed as a result of such capture, almost immediately (within $\sim 10^{-15}$ s) disintegrated by the emission of a proton of very high velocity. This was the process of nuclear transmutation brought about artificially with the help of $\alpha$-particles from a radioactive substance, leaving a residual nucleus of the isotope $^{17}\text{O}$ of oxygen. The process can be represented by means of an equation analogous to the equation for a chemical reaction as follows:

$$\frac{4}{2}\text{He} + \frac{14}{7}\text{N} \rightarrow \frac{18}{9}\text{F}^* \rightarrow \frac{17}{8}\text{O} + \frac{1}{1}\text{H}$$

...(10.1-1)

The intermediate step $^{18}\text{F}^*$ is known as a compound nucleus. (see later). It breaks up almost immediately after its formation. In writing such a nuclear reaction equation, we often omit this intermediate step and write only the initial and final steps in the process.

A nuclear reaction refers to a process which occurs when a nuclear particle (e.g., a nucleon, a nucleus or an elementary particle) comes into close contact with another during which energy and momentum exchanges take place. The final products of the reaction are again some nuclear particle or particles which leave the point of contact (reaction site) in different directions. The changes produced in a nuclear reaction usually involve strong nuclear force. Purely electromagnetic effects (e.g., Coulomb scattering) or processes involving weak interactions (e.g., $\beta$-decay) are usually excluded from the category of nuclear reaction. However, changes of nuclear states under the influence of electromagnetic interactions are included.

In general, a nuclear reaction can be represented by an equation in the following form:

$$\frac{2}{1}X + x \rightarrow \frac{2}{1}Y + y$$

or simply as $^{A}X(x,y)^{A}Y$.

Here X is the target nucleus which is bombarded by the projectile x. The resulting compound nucleus breaks up almost immediately by ejecting a particle y, leaving a residual nucleus Y. Since the chemical symbol of the atoms indicates their atomic numbers (Z), these are often omitted in writing the nuclear reaction equation. The projectile x and the emitted particle y are in many cases light nuclei such as protons ($p^+$), neutrons ($n$), deuterons ($d$). $\alpha$-particles ($\alpha$), $\gamma$-rays ($\gamma$) etc. and in the nuclear reaction equations, these symbols are generally used.

10.2 Types of nuclear reactions

The artificial transmutation of a nucleus produced in the pioneering experiment of Rutherford is a type of nuclear reaction. Various types of nuclear reactions have since been produced. These can be conveniently classified as below:

(a) Elastic scattering: In this case the ejected particle y is the same as the projectile x. It comes out with the same energy and angular momentum as x, so that the residual nucleus Y is the same as the target X and is left in the same state (ground state) as the latter. We can represent the process as $X(x,x)|X$. 

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(b) *Inelastic scattering*: In this case $y$ is the same as $x$. But it has different energy and angular momentum, so that the residual nucleus $Y (=X)$ is left in an excited state. The process can be written as $X(x, y)Y^*$, where the asterisk on $X$ indicates an excited state of $X$.

(c) *Radiative capture*: In this case the projectile $x$ is absorbed by the target nucleus $X$ to form the excited compound nucleus ($C^*$) which subsequently goes down to the ground state by the emission of one or more $\gamma$-ray quanta. We can write the process as $X(x, \gamma)Y$ ($Y = C$).

(d) *Disintegration process*: We can represent the process as $X(x, y) Y$ where $X, x, Y$ and $y$ are all different either in $Z$ or in $A$ or in both. The first nuclear transmutation observed by Rutherford is an example of this process: $^{14}\text{N}(\alpha, p)^{17}\text{O}$.

(e) *Many body reaction*: When the kinetic energy of the incident particle is high, two or more particles can come out of the compound nucleus. If $y_1, y_2, y_3$, etc. represent these different particles, we can write the reaction equation as $X(x, y_1, y_2, y_3, \ldots) Y$. Examples are $^{16}\text{O}(p, 2p)^{15}\text{N}$; $^{16}\text{O}(p, \alpha)^{12}\text{C}$; $^{16}\text{O}(p, 3p)^{14}\text{C}$ etc. When the energy of $x$ is very high, a very large number of reaction products usually result (3 to 20 for example). Such reactions are known as *spallation reactions*.

(f) *Photo-disintegration*: In this case the target nucleus is bombarded with very high energy $\gamma$-rays, so that it is raised to an excited state by the absorption of the latter. The compound nucleus $C^* = X^*$. The reaction can be written as $X(\gamma, \gamma) Y$.

(g) *Nuclear fission*: When $X$ is a heavy nucleus and $Y$, $Y$ have comparable masses, the reaction is known as nuclear fission. An example is $^{235}\text{U}(n, f)$. (see Ch. XIV).

(h) *Elementary particle reactions*: These involve either the production of elementary particles other than nucleons or nuclei as a result of the reaction or their use as projectiles or both of these. Examples are:

$$ p + p \rightarrow p + n + \pi^+; $$

$$ \pi^- + p \rightarrow \pi^0 + n; $$

$$ p + \pi^0 \rightarrow K^0 + \pi^0; $$

These reactions are usually produced at extremely high energies which may be several hundred MeV or more.

(i) *Heavy ion reactions*: In these reactions the target nucleus is bombarded by projectiles heavier than $\alpha$-particles. Various types of products may be produced. The reactions usually take place at fairly high energies (several hundred MeV) of the projectile. Examples are:

$^{10}\text{B}(^{16}\text{O}, ^{4}\text{He})^{22}\text{Na}$,

$^{14}\text{N}(^{14}\text{N}, ^{15}\text{N})^{23}\text{Na}$ etc.

10.3 *Conservation laws in nuclear reactions*

The occurrence of a nuclear reaction is usually governed by certain conservation laws.

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Nuclear Reactions (I)

(a) *Conservation of mass number*: The total number of neutrons and protons in the nuclei taking part in a nuclear reaction remains unchanged after the reaction. Thus in the reaction $X(x, y) Y$ represented by Eq. (10.1-2), the sum of mass numbers of $X$ and $x$ must be equal to the sum of the mass numbers of $Y$ and $y$:

$$ A + a = A' + a'; \quad \ldots(10.3-1) $$

In the general case of reactions involving elementary particles the law can be expressed by requiring the total number of heavy particles (baryons) remains unchanged in a reaction (see Ch. XVIII).

(b) *Conservation of atomic number*: The total number of protons of the nuclei taking part in a nuclear reaction remains unchanged after the reaction. This means that the sum of atomic numbers of $X$ and $x$ is equal to the sum of atomic numbers of $Y$ and $y$:

$$ Z + z = Z' + z'; \quad \ldots(10.3-2) $$

In view of the conservation law (a) and (b) above it is easily seen that the mass number and the atomic number of the product nucleus in Rutherford's experiment (Eq. 10.1-1) should be $A' = A + a - a' = 14 + 4 - 1 = 17$ and $Z' = Z + z - z' = 7 + 2 - 1 = 8$, so that the product nucleus must be the isotope $^{17}\text{O}$ of oxygen.

Further, in view of (a) and (b) the nucleon number $N$ remains unchanged in the reaction.

(c) *Conservation of energy*: The value of a nuclear reaction: In order to apply the law of conservation of energy in the case of a nuclear reaction, it is necessary to take into account the mass-energy equivalence predicted by the special theory of relativity. Conservation of energy requires that the total energy, including the rest-mass energies of all the nuclei taking part in a reaction and their kinetic energies, must be equal to the sum of the rest-mass energies and the kinetic energies of the products.

Writing $M_X, M_x,M_Y$ and $M_y$ as the rest-masses of the different atoms in Eq. (10.1-2), their rest mass energies are $M_{xc}^2, M_{x}^2, M_{yc}^2$ and $M_{y}^2$ respectively. Denoting the kinetic energy by $E$ we then get

$$ M_{xc}^2 + M_{x}^2 + E_x + E_x = M_{yc}^2 + M_{y}^2 + E_y + E_y $$

During the nuclear reaction, the target nucleus is usually at rest, so that $E_x = 0$. The above equation then becomes

$$ M_{xc}^2 + M_{x}^2 + E_x = M_{yc}^2 + M_{y}^2 + E_y + E_y \quad \ldots(10.3-3) $$

The above energy balance equation is often written without the factor $c^2$ in the mass-energy terms, which means that the masses are expressed in energy units.

It may be noted that though the nuclear masses are involved in a nuclear reaction, it is possible to write the energy–balance equation in terms of the atomic masses, since the electronic masses cancel out on the
two sides of the equation and the electronic binding energies can be neglected.

It may be noted that at relatively lower energies, the kinetic energy is given by the non-relativistic expressions: \( E = \frac{p^2}{2} \). When the energies of the particles involved in the reaction are very high, as in the case of many elementary particle reaction, the relativistic expression for the kinetic energy must be used: \( E = \sqrt{p^2 c^2 + M_0^2 c^4} - M_0 c^2 \). Here \( M_0 \) is the rest mass of the particle and \( p = M_0 v / \sqrt{1 - \beta^2} \) is its linear momentum (see Ch. XVIII).

\( d \) Conservation of linear momentum: If \( \mathbf{p}_X, \mathbf{p}_Y, \mathbf{p}_X \) and \( \mathbf{p}_y \) represent the momentum vectors of the different nuclei taking part in a reaction, then the law of conservation of linear momentum gives

\[
\mathbf{p}_X + \mathbf{p}_Y = \mathbf{p}_X + \mathbf{p}_Y \quad \ldots (10.3-4)
\]

Eq. (10.3-4) holds in an arbitrary frame of reference. In the laboratory frame of reference (L-system) in which the target nucleus is at rest \( p_x = 0 \) and the above equation becomes

\[
\mathbf{p}_X = \mathbf{p}_Y + \mathbf{p}_Y \quad \ldots (10.3-5)
\]

In the frame of reference in which the centre of mass of the two particles before collision is at rest (C-system), we have to write \( \mathbf{p}_X + \mathbf{p}_Y = 0 \), which gives \( \mathbf{p}_X + \mathbf{p}_Y = 0 \) i.e., the centre of mass of the product particles is also at rest in this system.

\( e \) Conservation of angular momentum: In a nuclear reaction of the type \( X + x \rightarrow Y + y \), the total angular momentum of the nuclei taking part in the reaction remains the same before and after the reaction.

Let \( I_X, I_Y, I_x \), and \( I_y \) denote the nuclear spins (total angular momentum) of the nuclei \( X, x, Y \), and \( y \) respectively. Let \( I_x \) represent the relative orbital angular momentum of \( X \) and \( x \) (i.e., in the initial state). Similarly, \( I_y \) denotes the relative orbital angular momentum of \( Y \) and \( y \) (i.e., in the final state). Then according to the law of conservation of angular momentum, we must have

\[
I_X + I_x + I_Y = I_X + I_y + I_y
\]

Application of the law of conservation of the angular momentum taking into account the well-known quantum mechanical properties of the former leads to certain selection rules.

\( f \) Conservation of parity: Since the nuclear reactions discussed in this chapter take place due to the strong interaction in which parity is conserved, the parity \( \Pi \), before the reaction must be equal to the parity \( \Pi \), after the reaction.

Denoting the intrinsic parities of the nuclei taking part in the reaction by \( \Pi_X, \Pi_y, \Pi_Y \), and \( \Pi_y \) we get for the initial and final states of the reaction

\[
\Pi_X \Pi_Y \Pi_y (-1)^i, \quad \Pi_Y \Pi_x (-1)^j
\]

The conservation of parity requires that

\[
\Pi_X \Pi_Y (-1)^i = \Pi_Y \Pi_y (-1)^j.
\]

Except in the cases of elementary particle reactions, the intrinsic parity need not be taken into account. Hence we get

\[
(-1)^i = (-1)^j.
\]

Parity conservation results in certain selection rules which limit the possible nuclear reactions that may occur starting from a given initial state \( i \). For example, in the case of elastic scattering \( l \) can change only by an even integer.

\( g \) Conservation of isotopic spin: Denoting the isotopic spin vectors for the initial and final states by \( T_i \) and \( T_f \), we have from the law of conservation of isotopic spin applicable in the case of strong interaction

\[
T_i = T_f
\]

Since for the reaction \( X + x \rightarrow Y + y \), \( T_i = T_X + T_x \) and \( T_f = T_Y + T_y \), we have

\[
T_X + T_x = T_Y + T_y
\]

Isotopic spin is a characteristic of the nuclear level. Hence the above conservation law can be used to identify the levels of the nuclei produced in the reaction. In particular if \( T_i = T_f = 0 \) (as for the deuterium or the \( \alpha \)-particle), we must have \( T_X = T_Y \).

This rule must be obeyed in reactions of the type \( (d, \alpha) \), \( (d, d) \), \( (d, \alpha) \), \( (\alpha, \alpha) \) etc. The rule has been verified for the nuclei \( ^{6}\text{Li} \), \( ^{14}\text{B} \) and \( ^{14}\text{N} \) for \( T = 0 \) in the ground states.

\subsection*{10.4 Collision between subatomic particles}

When a nuclear reaction takes place, certain conservation laws, including the laws of conservation of momentum and energy hold (see § 10.3). Without going into the details of the reaction mechanism, it is possible to deduce the energies and momenta of the particles produced in the reaction from simple kinematical considerations.

When a nuclear projectile is incident upon a nucleus (usually at rest), the particles may either suffer elastic collision or may produce a reaction in which new particles are usually produced. In elastic collision, no change in the internal states of the colliding particles takes place. It is therefore, essential to apply the laws of conservation of momentum and kinetic energy to analyse the kinematics of the collision process. In reactions, including inelastic collision, change in the internal state of the particles must be taken into account while applying the law of conservation of energy.

In the experimental arrangement, a beam of mono-energetic particles, called projectiles, is allowed to fall on the target containing the nuclei which are at rest.

The collision between a projectile and a target nucleus can be analysed from the point of view of an observer at rest in the laboratory. This is known as the laboratory frame of reference or the \( L \)-system. Alternatively, the collision may be analysed from the point of view of an observer at rest with respect to the centre of mass of the colliding particles, known as the \( C \)-system.
Elastic collision in L-system (non-relativistic):

Consider the elastic collision between a particle of mass $M_1$ and velocity $v_1$ (in the L-system) and a stationary target of mass $M_2$ ($v_2 = 0$). After the collision, the two particles fly apart from the point of collision with the velocities $v_1'$ and $v_2'$ at the angles $\theta_1$ and $\theta_2$, respectively, with respect to the incident direction.

Referring to Fig. 10.2, we get from the laws of conservation of momentum along and perpendicular to the incident direction:

$$p_1 = p_1' \cos \theta_1 + p_2' \cos \theta_2 \quad \text{(10.4.1)}$$
$$0 = p_1' \sin \theta_1 + p_2' \sin \theta_2 \quad \text{(10.4.2)}$$

Squaring and adding the above two equations we get

$$p_1'^2 = p_1^2 + p_2^2 - 2p_1p_2 \cos \theta_1 \quad \text{(10.4.3)}$$

We next apply the law of conservation of energy. Denoting the kinetic energies by $E$ we have

$$E = E_1' + E_2' \quad \text{(10.4.4)}$$

In terms of the momenta, we get

$$p_1^2 = \frac{p_1^2}{M_1} + \frac{p_2^2}{M_2} \quad \text{(10.4.5)}$$

Substituting for $p_2^2$ from Eq. (10.4.3), we get

$$p_1^2 \left(1 + \frac{M_2}{M_1}\right) - 2p_1p_1' \cos \theta_1 + p_1^2 \left(1 - \frac{M_2}{M_1}\right) = 0 \quad \text{(10.4.6)}$$

If we put $r = M_2/M_1$, the above equation becomes

$$p_1^2 (1 + r) - 2p_1p_1' \cos \theta_1 + p_1^2 (1 - r) = 0 \quad \text{(10.4.7)}$$

In terms of energies we get

$$E_1' (1 + r) - 2 \sqrt{E_1' E_1} \cos \theta_1 + E_1 (1 - r) = 0 \quad \text{(10.4.8)}$$

or,

$$E_1' (1 + r) - 2 \sqrt{E_1} E_1 \cos \theta_1 + (1 - r) = 0 \quad \text{(10.4.9)}$$

Eq. (10.4.8) is quadratic in $\sqrt{E_1}$.
Elastic collision in C system (non-relativistic):

We now consider the collision between two particles from the point of view of an observer at rest relative to the centre of mass C of the particles (Fig. 10.3). We shall denote the velocities and momenta in the C-system by the capital letters (V and P), while those in the L-system by the small letters (v and p). The energies and the angles of scattering will be denoted by E and θ in the C system and by ε and φ in the L system.

The particle M₁ is at rest in the L-system before collision (v₂ = 0). The velocity in the L-system of the centre of mass is

\[ v_c = \frac{M_1 v_1 + M_2 v_2}{M_1 + M_2} = \frac{M_1 v_1}{M_1 + M_2} \]  (10.4-15)

Hence the velocities of M₁ and M₂ in the C system before collision are respectively

\[ v_1 = v_1 - v_c = \frac{M_2 v_1}{M_1 + M_2} \]  (10.4-16)
\[ v_2 = v_2 - v_c = -\frac{M_1 v_1}{M_1 + M_2} \]  (10.4-17)

The corresponding momenta are

\[ P_1 = M_1 v_1 = \frac{M_1 M_2 v_1}{M_1 + M_2} = M_1 v_1 \]  (10.4-18)
\[ P_2 = M_2 v_2 = -\frac{M_1 M_2 v_1}{M_1 + M_2} = -M_1 v_1 \]  (10.4-19)

where \( \mu = M_1 M_2 / (M_1 + M_2) \) is the reduced mass. Thus the two particles have equal and opposite momenta before collision, so that their total momentum \( P_1 + P_2 = 0 \). Conservation of momentum then requires that the total momentum of the two particles after collision is also zero:

\[ P_1' + P_2' = P_1 + P_2 = 0 \]

We have denoted the momentum after collision by putting primes (') above the corresponding quantities before collision. Sums of the kinetic energies before and after collision are

\[ E_1 + E_2 = \frac{P_1^2}{2M_1} + \frac{P_2^2}{2M_2} = \frac{P^2}{2\mu} \]  (10.4-20)
\[ E_1' + E_2' = \frac{P_1'^2}{2M_1} + \frac{P_2'^2}{2M_2} = \frac{P'^2}{2\mu} \]  (10.4-21)

where \( |P_1| = |P_2| = |P| \) and \( |P_1'| = |P_2'| = |P'| \).

Since energy conservation requires that

\[ E_1' + E_2' = E_1 + E_2 \]

we get \( P' = P \) so that the magnitudes of momenta of the particles before and after collision are all equal:

\[ P_1 = P_2 = P_1' = P_2' \]  (10.4-22)

The momentum diagram of the particles is shown in Fig. 10.3. The two particles fly apart from the point of collision with equal and opposite momenta as shown so that \( \theta_1 + \theta_2 = \pi \).

The kinetic energy of the centre of mass is

\[ E_c = \frac{1}{2} \left( \frac{M_1 + M_2}{M_1 M_2} \right) v_c^2 = \frac{M_1}{M_1 + M_2} \epsilon_1 \]

...(10.4-23)

where \( \epsilon_1 = \frac{1}{2} M_1 v_1^2 \) is the kinetic energy of the incident particle in the L-system. The energy \( E_c \) given by Eq. (10.4-23) is not available for the production of any inelastic effect (e.g. reaction). The total amount of energy available for this purpose is

\[ \epsilon_1 - E_c = \frac{M_1}{M_1 + M_2} \epsilon_1 = \frac{M_2}{M_1 + M_2} \epsilon_1 = \frac{1}{2} \mu v_1^2 \]  (10.4-24)

From the above discussions, it is clear that there is no change in the kinetic energies and momenta of the particles after collision in the C system.

We now deduce the relationships between the angles of scattering in the two systems.

In Fig. 10.4 is shown the velocity diagram corresponding to the momentum diagram of Fig. 10.3. Since the magnitudes of the momenta remain unchanged by elastic collision in the C-system, the magnitudes of the velocities also remain unchanged. Hence \( V_1' = V_1 \), \( V_2' = V_2 \) as shown in Fig. 10.4.
The velocities \( v'_1 \) and \( v'_2 \) after collision of the two particles can then be easily obtained by the vector addition of the centre of mass velocity \( v_c \) (which remains unchanged after collision) with \( V'_1 \) and \( V'_2 \) respectively i.e.

\[
v'_1 = V'_1 + v_c, \quad v'_2 = V'_2 + v_c
\]

These are represented by the straight lines CB and CF respectively in Fig. 10.4.

Referring to the triangle CAB, we can write

\[
\frac{v_c}{\sin(\varphi_1 - \theta_1)} = \frac{V'_1}{\sin \theta_1} = \frac{V'_2}{\sin \theta_2}
\]

This gives the angle of scattering in the L-system

\[
\varphi = \theta_1 + \sin^{-1}\left(\frac{M_1}{M_2} \sin \theta_1\right)
\]

for \( M_1 = M_2, \varphi_1 = 2\theta_1 \)

Similar relationship can be obtained between \( \theta_2 \) and \( \varphi_2 \). Considering the triangle CDF, we have

\[
\frac{V'_2}{\sin \theta_2} = \frac{v_c}{\sin(\varphi_2 - \theta_2)}
\]

or since \( V'_2 = v_c \) we get

\[
\varphi_2 = 2\theta_2 \quad \text{...(10.4-26)}
\]

We then get

\[
\varphi_1 + \varphi_2 = \theta_1 + 2\theta_2 + \sin^{-1}\left(\frac{M_1}{M_2} \sin \theta_1\right) = \pi
\]

Hence

\[
\sin(\theta_1 + 2\theta_2) = \frac{M_1}{M_2} \sin \theta_1 \quad \text{...(10.4-27)}
\]

The velocity of the struck particle in the L-system is

\[
v''_2 = v_c \cos \theta_2 = \frac{2M_1 v'_1}{M_1 + M_2} \cos \theta_2 \quad \text{...(10.4-28)}
\]

**Special cases:**

(i) Let \( M_1 = M_2 \) as in the case of neutron-proton scattering. Eq. (10.4-25) gives \( \varphi_1 = 2\theta_1 \). Eq. (10.4-27) gives

\[
\theta_1 + 2\theta_2 = \pi - \theta_1
\]

or,

\[
\theta_1 + \theta_2 = \pi/2 \quad \text{...(10.4-29)}
\]

Thus in the L-system, the angle between the paths of the two particles of equal mass after collision is always 90°.

(ii) Let \( M_2 >> M_1 \). This is the case of scattering of a light particle like an electron from a very heavy particle (a nucleus). We have

\[
\theta_1 + 2\theta_2 = \pi
\]

which means

\[
\theta_2 = (\pi/2) - (\theta_1/2)
\]

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Also Eq. (11.4-28) gives \( v'_2 << v_1 \) i.e., the struck particle gets very little energy.

(iii) \( M_2 << M_1 \). This corresponds to the scattering of a very heavy particle (a nucleus) by a very light particle like an electron. Eq. (11.4-27) gives

\[
\sin \theta_1 = \frac{M_2}{M_1} \sin(\theta_1 + 2\theta_2) \rightarrow 0
\]

Hence \( \theta_1 = 0 \).

The incident particle goes on almost undeviated after collision. Note that this was one of the assumptions made in deriving the formula for the energy loss by heavy charged particle in passing through matter (see Ch. IV).

**Nonelastic collisions:**

Nuclear reactions including inelastic scattering (e.g. \( pp', nn' \)) belong to this type of collision. The particles produced after collision are usually different from those before collision. If \( M_3 \) and \( M_4 \) are the masses of the two particles produced by the reaction and their kinetic energies are \( E_3 \) and \( E_4 \), we can write the energy conservation equation as

\[
M_1 + M_2 + E_1 + E_2 = M_3 + M_4 + E_3 + E_4 \quad \text{...(10.4-30)}
\]

Here the masses are expressed in energy units (i.e. \( mc^2 \)). Writing

\[
Q = M_1 + M_2 - M_3 - M_4
\]

we then get (':: \( E_2 = 0 \))

\[
Q + E_1 = E_3 + E_4 \quad \text{...(10.4-31)}
\]

Eq. (10.4-31) along with the momentum conservation equation have to be used to find the energies of the reaction products. This will be discussed in more detail in § 10.5.

**10.5 Energetics of nuclear reactions**

During a nuclear reaction, energy is either evolved or absorbed. Reactions in which energy is evolved are known as *exoergic reactions* while those requiring absorption of energy are called *endoergic*. The total amount of energy evolved or absorbed during a nuclear reaction is called the \( Q \) value or simply the \( Q \) of the reaction. So by definition

\[
Q = E + E - E + E = E + E - E \quad \text{...(10.5-1)}
\]

if the target nucleus \( X \) is at rest.

\( Q \) is thus equal to the net surplus (or deficit) of the energies of the reaction products \( E + E \) over the energy supplied (\( E \)).

If the atomic masses are expressed in energy units, Eq. (10.3-3) can be rewritten as

\[
M_x + M_y + E_z = M_y + M_x + E_y + E
\]

Then we get from Eq. (10.5-1)

\[
Q = M_x + M_y - M_y - M_x \quad \text{...(10.5-2)}
\]
Since $Q \ll M_x$, we can neglect it in the denominator of Eq. (11.5-13). Also we can replace $M_y + M_y$ in the numerator by $M_y + M_x$. So we get finally

$$E_{th} = -Q \frac{M_y + M_x}{M_x} = -Q \left( 1 + \frac{M_x}{M_y} \right)$$

...(10.5-14)

So by measuring the minimum energy $E_y$ at which an exoergic reaction is initiated it is possible to determine the $Q$ value of the reaction.

An inspection of Eq. (10.5-10) shows that under certain circumstances $E_y$ will be a double-valued function of the projectile energy $E_x$, i.e., for a given $E_y$, there may be two values of $E_x$, the energy of the emitted particle. This happens only for endoergic reactions. The double valued nature of $E_x$ is revealed in Fig. 10.6 for the $^3$H(p, n) $^3$He endoergic reaction which has $Q = 1.7638$ MeV. Eq. (10.5-10) also shows that $E_y$ is single valued if the following condition is satisfied:

$$Q(M_y - M_x) \geq 0$$

or,

$$E_x \geq \frac{QM_y}{M_y - M_x}$$

$$E_x \geq \frac{QM_y}{M_y - M_x}$$

Fig. 10.6. $E_x$ versus $E_y$ graph in $^3$H(p, n) $^3$He reaction. Double valued nature of neutron energy should be noted.

Thus there is a limiting energy of the projectile above which the emitted particle energy will be single valued. This is given by

$$E'_x = \frac{QM_y}{M_y - M_x}$$

...(10.5-15)

For the case cited above $E'_x = 1.145$ MeV. For projectile energy greater than $E'_x$, the product particle $y$ can be emitted at all angles between $0^\circ$ and a maximum angle $\theta_{max}$, which can be found with the help of Eq. (10.5-10).

Exoergic reaction:

In this case, the reaction can occur for all values of $E_x$ including $E_x = 0$. For $E_x = 0$, the incident momentum is zero and hence the sum of the momenta of the product particles must be zero: $p_x + p_y = 0$. This means that $Y$ and $y$ proceed in opposite directions, so that $\theta + \varphi = \pi$. Also in this case $Q = E_y + E_x$.

In general Eq. (10.5-7) gives for $Q > 0$ only one value of $E_y$, the energy of the emitted particle, for a given $E_x$ and at a given angle of emission $\theta$. All values of $\theta$ are possible. Hence there is an energy distribution of the emitted particles between a maximum at $\theta = 0$ and a minimum at $\theta = \pi$. In the solution for Eq. (10.5-7) given by Eq. (10.5-9) the plus sign has to be chosen when a positive value of the momentum $p_y$ is to be obtained.

As we shall see in § 13.4c exploitation of the rigid correlation between $E_x$ and $\theta$ is the only way of obtaining monoenergetic neutron beams of different energies.

10.6 Experimental determination of $Q$

$Q$ of a reaction can be determined with the help of Eq. (10.5-1) by measurement of the energies $E_x$, $E_y$, and $E_T$ accurately. It can also be estimated from the precise values of the atomic masses of the nuclei taking part in the reaction, using Eq. (10.5-2). If one of the product nuclei (y) is a heavy particle, then it is usually difficult to measure its kinetic energy ($E_y$) accurately. However, it can be determined with the help of Eq. (10.5-6) from a knowledge of the masses and by measuring the energies $E_x$ and $E_T$ accurately. In this case the precise values of the masses need not be used. Instead the corresponding mass numbers will suffice.

If the emitted particle y is a charged particle, then one can use a scintillation counter, a proportional counter (gas-filled), a solid state counter or a magnetic spectrograph to determine its energy.

Scintillation spectrometers can be used when the resolving power needed is not very high. Since the charged particles have very small ranges in solids, the scintillation detector to be used can be quite thin for low energy reaction studies. The scintillator can be located close to the target to increase the solid angle which helps improve the statistics of counting. The resolving power is usually low, being ~ 20 to 30.

With solid state spectrometers, the resolving power is much better, being ~ 200 to 300. Since solid state detectors with sufficiently thick active layers are available now a days, one can go upto fairly high energies.

Magnetic spectrometers are by far the most suitable instruments for high resolution work. Resolving powers of the order of 1000 have been achieved with these instruments. Both single focusing and double focussing instruments have been developed. In the former, the particles emerging from a point of the median plane are focussed along a line...
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A highly versatile multigap magnetic spectrograph with nuclear emulsion plates as detectors has been used by H.A. Enge, and W.W. Buechner at the Massachusetts Institute of Technology (MIT) in the U.S.A. (see Rev. Sci. Instr. 34, 155, 1963). This instrument is actually twenty four instruments in one large vacuum chamber. Both energy and angular distributions of the particles are recorded simultaneously. Energy distributions at different angles θ at the interval of 7.5° are obtained.

After exposure, the plates are developed and scanned under microscopes. The number of tracks of the particles having the correct length and direction are counted. At each exposure lasting for 1 - 10 hr the number of data points obtained is 36000.

Fig. 10.7 shows a typical spectrum obtained with this instrument for the reaction ^47Sc (d, p) ^46Sc using a deuteron beam accelerated in the 8 MeV Van de Graaff generator. The different peaks correspond to different states of the residual nucleus ^46Sc.

Differential cross sections dσ/dΩ for the peaks at different energies as functions of the angle of emission were also determined. These show pronounced maxima in the forward direction.

The positions of the maxima are determined by the orbital angular momenta of the states in which the neutron is captured in the (d, p) reaction.

10.7 Cockroft and Walton’s experiment on nuclear transmutation by artificially accelerated projectiles

In the early years after Rutherford’s discovery of artificial transmutation of nuclei, nuclear reactions were produced by using the high energy α-particles from naturally radioactive substances, since these were the only projectiles available at that time. However due to limitations of energy and intensity, need for other sources of projectiles was keenly felt. Another limitation was that the α-particles being doubly charged were very strongly repelled by the positive charge of the nucleus and hence could not penetrate the heavier nuclei to produce nuclear reactions. J.D. Cockroft and E.T.S. Walton, two associates of Rutherford in 1932 developed a charged particle accelerator with the help of which a beam of protons could be accelerated to high energy. With the help of this high energy beam of protons they were able to produce disintegration of the nucleus ^7Li and studied the following reaction:

^7Li + ^1H → ^3He + ^2He

The accelerator developed by Cockroft and Walton is known as the voltage multiplier or simply as Cockroft Walton generator (see Ch. XII).

In their initial experiments, they could produce about 700,000 volts. Protons accelerated through this voltage gained a kinetic energy of 0.7 MeV. Actually the above nuclear reaction could be produced by protons accelerated to as low an energy as 0.15 MeV, even though the potential barrier at the nuclear surface of ^7Li was about 1.5 MeV high for protons (V = Ze²/A π ε₀ R).
This was possible because there was a finite probability of the protons penetrating through the potential barrier as determined by Gamow's theory (see Ch. IV).

The intensity of the accelerated proton beam was much higher than was available from natural α-sources. For example, 1 g of $^{226}$Ra emits about $3.7 \times 10^{10}$ α-particles per second in all directions (4π solid angle) so that the number of particles falling on a metal foil of 1 cm$^2$ area at a distance of 1 cm from the source is of the order of $10^9$ per second. On the other hand, it is quite easy to get a proton current of 1 microampere from the accelerator, which is equivalent to $6.25 \times 10^{15}$ protons per second falling on a target.

Cockcroft and Walton's experimental arrangement is shown in Fig. 10.8. T is the lithium target from which α-particles were found to be emitted when the target was bombarded by the proton beam. The α-particles were detected by observing the scintillations produced by them on a ZnS screen S with the help of a microscope M. The range of the α-particles could be measured with the help of the absorber foils A.

Later P.J. Dec. and E.T.S. Walton produced the above reaction within a cloud chamber with the help of a 0.25 MeV proton beam and obtained the photograph of the two α-particles produced in the reaction. As can be seen from their cloud chamber photograph (Fig. 10.9) the two α-particles are ejected in opposite directions from the target. Measurement of their range showed that they had the energy of 8.6 MeV each. From these energy values the $Q$ of the reaction was found to be 16.95 MeV. Later

\[
p = \sqrt{2M_p E_\alpha} = \left[2 \times 4 \times 6.66 \times 10^{-27} \times 8.6 \times 1.6 \times 10^{-15}\right]^{1/2}
\]

\[
= 13.5 \times 10^{-20} \text{kg m/s}
\]

On the other hand, the momentum of a proton of energy 0.25 MeV is

\[
p_p = \sqrt{2M_p E_p} = \left[2 \times 4 \times 6.66 \times 10^{-27} \times 0.25 \times 1.6 \times 10^{-13}\right]^{1/2}
\]

\[
= 1.15 \times 10^{-20} \text{kg m/s}
\]

Thus the emitted α-particles have much higher momenta each than the incident proton. Conservation of momentum is possible in this case if the two α-particles proceed in almost opposite directions (see Fig. 10.10).

Fig. 10.9. Disintegration of $^7$Li nucleus by proton bombardment observed in the cloud chamber photograph of Dec. and Walton.

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Nuclear Reactions (I)

more accurate measurement gave $Q = 17.33$ MeV, which agreed well with the value obtained from the atomic mass differences of the nuclei involved.

That the two α-particles in Cockcroft and Walton's experiment should be emitted in opposite directions can be understood from momentum conservation. An α-particle of energy 8.6 MeV has a momentum

\[
p = \sqrt{2M_p E_\alpha} = \left[2 \times 4 \times 6.66 \times 10^{-27} \times 8.6 \times 1.6 \times 10^{-15}\right]^{1/2}
\]

\[
= 13.5 \times 10^{-20} \text{kg m/s}
\]

On the other hand, the momentum of a proton of energy 0.25 MeV is

\[
p_p = \sqrt{2M_p E_p} = \left[2 \times 4 \times 6.66 \times 10^{-27} \times 0.25 \times 1.6 \times 10^{-13}\right]^{1/2}
\]

\[
= 1.15 \times 10^{-20} \text{kg m/s}
\]

Thus the emitted α-particles have much higher momenta each than the incident proton. Conservation of momentum is possible in this case if the two α-particles proceed in almost opposite directions (see Fig. 10.10).

Fig. 10.10. Momentum conservation in Cockcroft-Walton's experiment.

It may be noted that Cockcroft and Walton's experiment provided the first direct experimental verification of Einstein's mass-energy equivalence principle.

The voltage multiplier developed by Cockcroft and Walton can generate voltages up to a maximum of about $10^8$ volts which can accelerate protons to about 1 MeV and α-particles to about 2 MeV. These particles can disintegrate lighter nuclei. For the disintegration of heavier nuclei with charged particle beams, much higher energies are required. Also for producing endoergic reactions, higher energy projectiles are needed. To meet these ends, various types of charged particle accelerators have been developed which can accelerate the particles to extremely high energies. For nuclear reaction studies energies up to a few hundred MeV are sufficient. However, for elementary particle experiments, beams of charged particles accelerated to thousands of MeV ($10^9$ eV) are needed.

In Ch. XII, we shall describe some of these different types of accelerators.
10.8 Cross section of nuclear reaction

The probability of the occurrence of a nuclear reaction is measured by the reaction cross section. It is usually designated by the symbol \( \sigma \). The cross section of a nuclear reaction \( X(x, y) Y \) can be written as \( \sigma(x, y) \). If a parallel beam of \( N \) projectiles is incident in a given interval of time upon a target foil of thickness \( \Delta x \) and surface area \( S \) normally, then the number of nuclei in \( T \) undergoing transformation due to the reaction of the type under consideration, is proportional to the intensity of the incident beam of projectiles and to the total number of target nuclei present in the foil (see Fig. 10.11(a)). The incident particle intensity is \( (N/S) \) and the number of nuclei present in the foil is \( (n S \Delta x) \). So the number of nuclei transformed is

\[
\Delta N \propto (N/S) (n S \Delta x)
\]

or,

\[
\Delta N = \sigma N n \Delta x = \sigma N n_1...
\]

(10.8-1)

Fig. 10.11. (a) Bombardment of target foil (T) by a beam of particles.
(b) Geometrical significance of reaction cross section.

Here \( n_1 = n \Delta x \) is the number of target nuclei per unit area of the foil, \( n \) being the number of nuclei per unit volume. Eq. (10.8-1) shows that since both \( \Delta N \) and \( N \) are pure numbers and \( n_1 = n \Delta x \) has the dimension of the reciprocal of an area, \( \sigma \) has the dimension of an area. Hence it is called the cross section and measures the probability of the occurrence of the reaction when a single particle (\( N = 1 \)) falls on a single target nucleus present per unit area (\( n_1 = 1 \)). Since the nuclear radii are of the order of \( 10^{-14} \) to \( 10^{-15} \) m, the cross section of the nuclear reaction is of the order of \( 10^{-28} \) m\(^2\). The commonly used unit of the nuclear reaction cross section is a barn:

\[
1 \text{ barn} = 10^{-28} \text{ m}^2
\]

Nuclear Reactions (1)

Though the cross sections for most nuclear reactions are of the order of a few barns or even less, they may be very high (several thousand barns) for some special types of reactions, such as the \((n, \gamma)\) reaction induced by thermal neutrons or the neutron-induced resonance reactions (see Ch. XI).

The geometrical significance of the reaction cross section can be understood in the following manner. Referring to Fig. 10.11b we see that if \( R \) is the effective radius of the target nucleus for a given reaction, then the projection of its surface area on a plane perpendicular to the direction of motion of the projectile, shown shaded in the figure is \( \pi R^2 \). So the number of projectiles encountering each target nucleus is \( \pi R^2 N_1 \), where \( N_1 = N/S \) is the number of projectiles incident per unit area of the target.

The projectiles are assumed to be mass-points. Since there are \( n_1 \) nuclei per unit area of the target, the number of projectiles intercepted by the target nuclei in the foil is

\[
n_1 S \times \pi R^2 N_1 = \pi R^2 N_1...
\]

(10.8-2)

where \( N = N_1 \times S \) is the total number of projectiles incident on the target. Hence the probability of encounter between a single projectile (\( N = 1 \)) with one nucleus per unit area (\( n_1 = 1 \)) in the target foil is

\[
\frac{\pi R^2 N_1}{n_1} = \pi R^2 N = \pi R^2...
\]

(10.8-3)

Actually the probability \( \sigma \) of encounter between a single projectile and a single target nucleus per unit area is not determined by \( \pi R^2 \) alone. This probability depends on the nature of the interaction between the projectile and the target nucleus, the energy of the projectile and other factors. Besides, the incident particle is not a mass-point as assumed above. So the reaction cross section depends on its size also. For very low energy projectiles, the de Broglie wavelength \( \lambda = h/p \) is much longer than their geometrical extension, so that the region over which they interact is much larger than their geometrical cross section. This is the reason for the cross section of the \((n, \gamma)\) reaction with thermal neutrons to be usually very large as stated above.

In the case of charged particles, the cross section is considerably reduced because of the strong electrostatic repulsion of the target nucleus.

In the above discussions, it has been assumed that the total projected area of all the nuclei in the foil which is \( (\pi R^2 n_1 S) \) is small compared to the area \( S \) of the foil. This is true only if the foil thickness is small.

10.9 Partial cross sections

When a nuclear projectile \( x \) is absorbed by a target nucleus \( ^A X \), a very short lived compound nucleus is formed (see § 10.1) which can break
up by the emission of different types of nuclear particles (y), leaving a different residual nucleus (Y) in each case (see §11.4). Thus we may have reactions of the type \( X(x, y)Y \). \( X(x, y)Y' \), \( X(x, y')Y' \) etc. In addition, we may have elastic and inelastic scatterings. Each of these different reactions induced by the same projectile \( x \) in the same target nucleus \( X \) has a different cross section, e.g., \( \sigma(x, y) \), \( \sigma(x, y') \), \( \sigma(x, y'') \) etc. in addition to \( \sigma(x, x) \) and \( \sigma(x, x') \). The total cross sections for the interaction of \( x \) with \( X \) for a given energy \( E \) of \( x \) can be written as

\[
\sigma_i = \sigma_e + \sigma_r = \sigma(x, x) + \sigma(x, x') + \sigma(x, y) + \sigma(x, y') + \sigma(x, y'') + \ldots
\]

...(10.9.1)

The cross sections for the individual types of reactions are known as partial cross sections. \( \sigma_e \) gives the sum of the partial cross sections for all non-elastic processes including inelastic scattering for which the partial cross section is \( \sigma(x, x') \).

\[
\sigma_r = \sigma(x, x') + \sigma(x, y) + \sigma(x, y') + \sigma(x, y'') + \ldots
\]

...(10.9.2)

It is usually called the reaction cross section to distinguish from the elastic scattering cross section \( \sigma_e = \sigma_{el} \).

Reaction cross sections are often expressed in terms of the reaction channels, which are specified in terms of the energy, angular momentum and radius. A nuclear reaction can be written as

\[ X + x \rightarrow C \rightarrow Y + y \]

For a given \( E \), the l.h.s. of this equation viz., \( X + x \), is known as the entrance channel. (Here it has been assumed that the target nucleus \( X \) is at rest). The right hand side giving the final products constitutes the exit channel. For elastic scattering the exit channel is identical with the entrance channel.

A precise definition of a channel may be given by stating that a channel is a possible pair of product nuclei, each in a definite quantum state.

Though the reaction (10.1-2) is not the most general reaction (which may involve many particle emission processes), it is sufficiently general to include most of the known nuclear reactions at low energies. There is one exception, viz., the radiative capture process i.e., \( \sigma(x, y) \) in which X and x stay together to form the nucleus C' with the emission of a \( \gamma \)-ray due to the deexcitation of the compound nucleus C'.

The total cross section \( \sigma = \sigma_r + \sigma_{el} \) determines the absorption coefficient for the beam of particles incident on the target foil. Using Eq. (10.8-1), we can write for a target foil having \( N \) nuclei per unit volume of infinitesimal thickness \( dx \) on which a particle beam of intensity \( N_i = N/S \) is incident perpendicularly as

\[
dn_i = -\sigma_i n_i dx
\]

...(10.9.3)

where the minus sign is introduced on the r.h.s. to indicate the diminution in the beam intensity as it comes out of the foil. Integration gives for a foil of finite thickness \( x \)

\[
n_r = n_0 \exp(-\mu x)
\]

...(10.9.4)

where

\[
\mu = \sigma_i n
\]

...(10.9.5)

is the total absorption coefficient. Here \( n_0 \) is the intensity of the beam incident on the foil and \( n_r \) is the emergent intensity. By measuring \( n_r \), it is possible to determine the total cross section \( \sigma_r \) with the help of Eqs. (10.9.4) and (11.9-5) if the incident intensity \( n_0 \) is known.

10.10 Reaction yield

The number of product nuclei Y produced as a result of the reaction \( X(x, y) \) gives the yield of the reaction. If Y is stable, its number goes on increasing linearly with time. The number of Y nuclei produced in time \( dt \) is equal to the number of X nuclei transmuted as a result of the above reaction. If \( \sigma(x, y) \) denotes the cross section, we can write

\[
dN_r = \sigma(x, y)N_0 n_i dt
\]

...(10.10.1)

where \( n_i \) is the number of projectiles incident on the target foil per second per unit area and \( N_0 \) is the total number of target nuclei in the foil. Then in time \( t \), the number of Y nuclei produced is

\[
N_r = \sigma(x, y)N_0 n_i t
\]

...(10.10.2)

If the product nuclei Y are radioactive with a disintegration constant \( \lambda \), then the rate of change of the number of Y nuclei is equal to the difference between the rate of its production \( \sigma(x, y)N_0 n_i \) and the rate of its disintegration \( \lambda N_r \). So we can write

\[
\frac{dN_r}{dt} = \sigma(x, y)N_0 n_i - \lambda N_r
\]

...(10.10.3)

or,

\[
\frac{dN_r}{N_r - \sigma(x, y)N_0 n_i/\lambda} = -\lambda dt
\]

Integration gives

\[
N_r = \sigma(x, y)N_0 n_i/\lambda + \lambda A \exp(-\lambda t)
\]

A is the integration constant. If \( N_r = 0 \) at \( t = 0 \), then we get

\[
A = \sigma(x, y)N_0 n_i/\lambda
\]

So we have finally

\[
N_r(t) = \frac{\sigma(x, y)N_0 n_i}{\lambda} [1 - \exp(-\lambda t)]
\]

...(10.10.4)

Eq. (10.10.4) shows that \( N_r(t) \) increases exponentially till it reaches a saturation value after a long time \( t \rightarrow \infty \) from the start of the bombardment:

\[
N_r = \frac{\sigma(x, y)N_0 n_i}{\lambda}
\]

...(10.10.5)
In practice $N_y$ becomes almost equal to the saturation value after ten or twelve half lives. $N_x$ becomes larger for larger values of $a(x,y)$. Since $a(x,y)$ is larger for slow neutrons (see later), the saturation yield $N_s$ of the product nuclei can be increased by using slow neutrons as projectiles. Further $N_x$ becomes larger for smaller values of $\lambda$, i.e., for longer lived product nuclei. Finally $N_s$ can be increased by increasing the intensity of the incident beam of projectiles ($n_i = N/s$). The saturation yield of short half-lived isotopes (typical few seconds to few days) by thermal neutron bombardment within a nuclear reactor is usually of the order of $10^{12}$ to $10^{17}$ nuclei ($10^4$ to $10^5$). For a long half-lived isotope (e.g., $^{239}$Pu) with a half-life of the order of $10^7$ years, the yield may be of the order of kilogramme.

Measurement of the yield of the product nucleus $Y$ gives the value of the cross section of the reaction $a(x,y)$.

The number of $Y$ atoms left in the sample after a time $\tau$ from the end of the bombardment of the foil is given by

$$N_Y(\tau) = N_Y(t) \exp(-\lambda \tau)$$

$$= \frac{\sigma(x,y) N_0 n_i}{\lambda} (1 - \exp(-\lambda \tau)) \exp(-\lambda \tau)$$  \hspace{1cm} (10.10-6)

Its radioactivity at this instant is

$$\frac{dN_Y}{dt} = \lambda N_Y(\tau) = \frac{\sigma(x,y) N_0 n_i}{\lambda} (-\exp(-\lambda \tau)) \exp(-\lambda \tau)$$  \hspace{1cm} (10.10-7)

If the bombardment lasts for a very long time ($t \to \infty$) so that the number of $Y$ atoms reaches saturation value, we get at the time $\tau$ after the stoppage of bombardment

$$N_Y(\tau) = \frac{\sigma(x,y) N_0 n_i}{\lambda} \exp(-\lambda \tau)$$  \hspace{1cm} (10.10-8)

and

$$\frac{dN_Y}{dt} = \frac{\sigma(x,y) N_0 n_i}{\lambda} \exp(-\lambda \tau)$$  \hspace{1cm} (10.10-9)

10.11 Reaction induced by $\alpha$-particles

When a nuclear reaction is induced by an $\alpha$-particle, the compound nucleus may break up by the emission of a proton, a neutron, a $\gamma$-ray photon etc.

($\alpha, p$) reaction: In § 10.1 we discussed about the first nuclear transmutation produced by Rutherford by bombarding nitrogen nucleus with $\alpha$-particles from a naturally radioactive substance (Eq. 10.10-1). This type of reaction is called an ($\alpha, p$) reaction. In general such a reaction can be represented by the following equation.

$$^4 \text{He} + ^{12} \text{C} \rightarrow ^{16} \text{O} + ^1 \text{H}$$

Using Eq. (10.5-3), the $Q$ of this reaction can be written as

$$Q = B_p - B_x - B_\alpha = (A + 3) f_{BY} - N_{BX} - 4 f_{\beta}\alpha$$
accelerators. Such α-particle beams are extensively used in the study of α-induced nuclear reactions.

(α, γ) reaction: This type of reaction, also known as the radiative capture of an α-particle, has been observed in some cases, e.g., 7Li (α, γ) 11B. The general formula is

$$\frac{1}{2}X + \frac{1}{2}He \rightarrow \frac{1}{2}He + \frac{1}{2}He$$

These reactions are usually exoergic.

More than one particle emission: For high α-energy, more than one particle may be emitted from the compound nucleus, producing such reactions as (α, 2n), (α, 2p), (α, 3n) etc.

10.12 Discovery of induced radioactivity

In 1933, Irene Curie-Joliot and her husband Frederic Joliot in Paris discovered induced radioactivity. They bombarded an aluminium foil with α-particles from a naturally radioactive substance (polonium) and observed the emission of neutrons in the process. They also found that positrons were emitted at the same time and that the emission of the positrons continued for some time even after the stoppage of the bombardment of the foil by α-particles. The intensity of the positrons was found to decrease with time exponentially.

To interpret their results, the Jolios assumed that the bombardment of aluminium by α-particles led to the production of the isotope 30P by 27Al(α, n) reaction. They further assumed that the residual nucleus 30P produced in the reaction was radioactive and decayed by positron emission:

$$^{30}P \beta^+ \rightarrow ^{30}Si$$

In order to substantiate their conclusion, they verified the chemical nature of the new radioactive product by separating it from the target by standard radiochemical methods and showed that the positron emission took place from the separated phosphorus. The phenomenon is known as induced or artificial radioactivity. The half-life of 30P is 2.55 min.

Similar phenomenon was observed by the Jolios with boron and magnesium. In each case the nature of the radioactive product was established by chemical separation.

The discovery of induced radioactivity is of great importance and the Jolios were jointly awarded Nobel prize in 1935 for it. Most products of artificial transmutation of elements are radioactive. They decay mainly by β− or β+ emission or by orbital electron capture. In the case of some heavy elements they are found to decay by α-emission or by spontaneous fissions. The artificial radioelements are extensively used for research in physics, chemistry, agricultural science, physiology and medical science.

10.13 Proton induced reactions

High energy proton beams are available from particle accelerators by accelerating hydrogen ions.

When a high energy proton falls on a target nucleus, the compound nucleus that is formed may disintegrate by the emission of different types of nuclear particles, e.g., proton, neutron, deuteron, α-particle, γ-ray etc.

Nuclear Reactions (I)

In the first case, we get elastic or inelastic scattering while in the other cases we get nuclear transmutation.

(p, α) reaction:

Out of these, we have discussed about the (p, α) reaction on 7Li first produced by Cockcroft and Walton (see § 10.7). From Eq. (10.5-3) it can be shown that the (p, α) reaction is usually exoergic. Writing the general formula for a (p, α) reaction as

$$\frac{1}{2}X + \frac{1}{2}H \rightarrow \frac{1}{2}He + \frac{1}{2}He$$

we get

$$Q(p, α) = (A - 3)E_{p} + 4E_{He} - AE_{He}$$

assuming $E_{p} = 8$ MeV for medium heavy nuclei. For target nuclei with lower values of $A$, $Q$ may be much higher.

Some examples of (p, α) reaction are given below:

- $^5Li + \frac{1}{2}H \rightarrow ^4Be + ^3He + ^2He$ (Q = 4 MeV)
- $^5Li + \frac{1}{2}H \rightarrow ^4Be + ^4He$ (Q = 17.35 MeV)
- $^{15}B + \frac{1}{2}H \rightarrow ^{12}C^* + ^2He$ (Q = 8.59 MeV)
- $^{19}F + \frac{1}{2}H \rightarrow ^{16}O + ^2He$ (Q = 8.12 MeV)
- $^{23}Na + \frac{1}{2}H \rightarrow ^{20}Mg^* + ^4He$ (Q = 2.38 MeV)
- $^{63}Cu^* + \frac{1}{2}H \rightarrow ^{60}Zn + ^2Ni + ^2He$ (Q = 3.76 MeV)

Out of these, the residual nucleus 4Be formed in the third reaction is highly unstable. It breaks up almost immediately after its production into two α-particles (2Be + 4He). Thus the final products of this reaction are three α-particles.

(p, n) reaction:

The general formula for such a reaction is

$$\frac{1}{2}X + \frac{1}{2}H \rightarrow \frac{1}{2}He + \frac{1}{2}He$$

In this case the residual nucleus Y is isobaric (same A) with the target nucleus with the atomic number one unit higher. Since two isotopes differing in Z by one unit cannot both be stable (see Ch. IX), the residual nucleus $^{A+1}Y$ must be β-active, the target nucleus being necessarily stable. Because of its higher Z, it will decay by β− emission (or by electron capture) into the $^{A}X$.

Some examples of (p, n) reactions are

- $^{15}N + \frac{1}{2}H \rightarrow ^{12}C^* + ^1C + ^1n$ (Q = −1.763 MeV)
- $^{23}Na + \frac{1}{2}H \rightarrow ^{24}Mg^* + ^1n$ (Q = −4.84 MeV)
from the compound nucleus to produce reactions like \((p, 2n), (p, pn), (p, 2p), (p, 3n)\) etc.

10.14 Deuteron induced reactions

Deuteron is the nucleus of the heavy hydrogen or the deuterium atom. It is an isotope of hydrogen of mass number 2 which is present in natural hydrogen with a relative abundance of 0.015%. Deuterium is obtained by repeated electrolysis of water. Ordinary water \((H_2O)\) is always mixed with a small fixed proportion of heavy water \((D_2O)\) in all natural sources of water. During electrolysis, light water \((H_2O)\) electrolizes faster than heavy water. As a result, if the electrolysis is carried out for a very long time, the proportion of heavy water in the residue that is left becomes higher. Repeated electrolysis ultimately yields almost pure heavy water. Huge quantity of electrical energy is required for such separation of \(D_2O\) from ordinary water. To get \(10^3\) kg \(1\) g of pure heavy water, about 30,000 ampere-hours of electrical energy is required. In India, heavy water plants have been set up at Nagal, Rana Pratap Sagar, Baroda and Tutikorin. Large quantities of heavy water are required in nuclear reactors as moderator (see Ch. XV).

Heavy water has the same chemical properties as ordinary water. But its physical properties are considerably different. Its specific gravity is 1.108.

Electrolysis of pure heavy water yields the heavy hydrogen atom which when ionized gives the heavy hydrogen nucleus or the deuteron \((d)\) within which a proton and a neutron are bound together with a binding energy of 2.226 MeV. The deuterons can be accelerated to high energies by particle accelerators like the protons and can be used as projectile to induce different types of nuclear reactions. During such reactions, protons, neutrons, \(\alpha\)-particles etc. may be emitted.

\((d, \alpha)\) reactions:

This type of reaction can be represented by the general formula:

\[
\frac{1}{2} X + \frac{1}{2} H \rightarrow \frac{1}{2} X^+ + \frac{1}{2} \alpha \rightarrow \frac{1}{2} A + \frac{1}{2} \alpha + \frac{1}{2} \alpha
\]

This reaction is usually exoergic as can be seen easily. For a medium heavy nucleus,

\[
Q(d, \alpha) = B_d + B_n - B_X - B_d = (A - 2) f_{\alpha} + 28 - A f_{\alpha} - 2.2
\]

Thus \(Q(d, \alpha) > 0\). Some examples of \((d, \alpha)\) reactions are given below:

\[
\begin{align*}
\frac{6}{3} Li + \frac{1}{2} H & \rightarrow \frac{8}{4} Be^+ \rightarrow \frac{6}{2} He + \frac{4}{2} He \quad (Q = 22.4 \text{ MeV}) \\
\frac{14}{7} N + \frac{1}{2} H & \rightarrow \frac{15}{7} O^+ \rightarrow \frac{12}{6} C + \frac{4}{2} He \quad (Q = 13.57 \text{ MeV}) \\
\frac{23}{11} Na + \frac{1}{2} H & \rightarrow \frac{25}{12} Mg^+ \rightarrow \frac{21}{10} Ne + \frac{12}{6} He \quad (Q = 6.9 \text{ MeV})
\end{align*}
\]
The alternative mechanism is known as stripping process. It belongs to a class of reactions known as direct reactions. We shall discuss about it in Ch. XI.

Both \((d, p)\) and \((d, n)\) reactions are observed in the deuteron-deuteron collision:

\[
Z + 1^7 H \rightarrow 2^1 H \rightarrow 4^2 He^* \rightarrow 3^0 He + 1^1 H \quad (Q = 4.03 \text{ MeV})
\]

\[
Z + 1^7 H \rightarrow 4^2 He^* \rightarrow 3^0 He + 1^0 n \quad (Q = 3.26 \text{ MeV})
\]

The product atom \(^3\)H in the first case is known as triton and is an isotope of hydrogen of mass number 3. Its nucleus is called triton. It is \(\beta\)-active with end point energy 0.019 MeV.

\[
3^0 H \rightarrow 3^1 He \quad (\tau = 12.4 \text{ y})
\]

The product \(^3\)He in the second of the above reactions is stable. It is an isotope of helium of mass number 3. It is present in natural helium with relative abundance of \(1.4 \times 10^{-4}\%\).

If sufficient quantity of tritium is generated by the above or some other nuclear reaction, then the tritium may be bombarded with deuterons to produce the following \((d, n)\) reaction:

\[
Z + 1^7 H \rightarrow 2^1 H \rightarrow 4^2 He^* \rightarrow 3^0 He + 1^0 n \quad (Q = 17.6 \text{ MeV})
\]

All the three reactions discussed above are known as nuclear fusion and are of great importance in the energy release by thermonuclear process (See Ch. XV).

\((d, t)\) reactions: The general formula for this type of reaction is as follows:

\[
\frac{4}{2} X + 1^7 H \rightarrow 4^2 + 2^1 X^* \rightarrow 4^0 X + 1^1 H
\]

The product nucleus \(X\) is an isotope of the target \(X\) with mass number one unit lower. The cross section of this type of reaction is rather low. Following are a few examples:

\[
\frac{7}{3} Li + 1^7 H \rightarrow 9^2 Be^* \rightarrow 8^1 Be + 0^1 n \quad (Q = -0.996 \text{ MeV})
\]

\[
\frac{9}{4} Be + 1^7 H \rightarrow 13^1 B^* \rightarrow 12^0 B + 0^1 n \quad (Q = -1.625 \text{ MeV})
\]

\[
\frac{35}{17} Cl + 1^7 H \rightarrow 37^{36} Ar^* \rightarrow 36^{38} Ar + 0^1 n \quad (Q = 6.28 \text{ MeV})
\]

More than one particle emission:

At higher energies of the deuteron \((E_d > 20 \text{ MeV})\), the reactions \((d, 2n)\), \((d, 2p)\), \((d, 3n)\) etc. in which two or more particles are emitted from the compound nucleus gain importance.

10.15 Neutron-induced reactions

Since the discovery of neutrons by Sir James Chadwick (see §13.1) they have been used extensively in producing nuclear reactions.

Neutrons are electrically neutral. Hence their use as projectiles to induce nuclear transmutations is of special advantage as they are not repelled by the electric charge of the target nucleus. Thus even a zero energy neutron can enter into a nucleus, however high its atomic number.
may be. These zero energy neutrons can produce exoergic reactions. For
endoeergic reactions, the incident neutrons must have kinetic energy
greater that the reaction threshold.

To produce nuclear reactions with neutrons, it is necessary to have
high intensity neutron sources. Neutrons are produced in nuclear
reactions. Those reactions for which the cross sections are high, are
specially suitable for use as neutron sources. We shall discuss about some
of the commonly used neutron sources in Ch. XIII.

Nuclear reactions induced by neutrons are associated with the
emission of α-particles, protons, γ-rays, deuterons etc.

(n, α) reactions:
The general formula is
\[ \frac{A}{2} X + \frac{1}{2} n \rightarrow \frac{A-1}{2} C^* \rightarrow \frac{A-3}{2} Y + \frac{3}{2} \text{He} \]
Some examples are:
\[ \frac{7}{3} \text{Li} + \frac{1}{2} n \rightarrow \frac{3}{3} \text{Li}^* \rightarrow \frac{1}{3} \text{H} + \frac{1}{2} \text{He} \quad (Q = 4.785 \text{ MeV}) \]
\[ \frac{10}{5} \text{B} + \frac{1}{2} n \rightarrow \frac{7}{5} \text{B}^* \rightarrow \frac{1}{5} \text{Li} + \frac{3}{5} \text{He} \quad (Q = 2.79 \text{ MeV}) \]
\[ 17 \text{Cl} + \frac{1}{2} n \rightarrow \frac{16}{17} \text{Cl}^* \rightarrow \frac{32}{15} \text{P} + \frac{3}{2} \text{He} \quad (Q = 0.935 \text{ MeV}) \]
Out of these, the first two reactions have fairly large cross sections. So
they are utilized in the construction of neutron detectors (see Ch. XIII).

(n, α) reactions are usually exoergic, specially for medium heavy
nuclei.

(n, p) reaction:
The general formula for this type reaction is
\[ \frac{A}{2} X + \frac{1}{2} n \rightarrow \frac{A+1}{2} C^* \rightarrow \frac{A}{2} Y + \frac{1}{2} \text{H} \]
The product nucleus Y is an isobar of this target nucleus X with
atomic number one unit lower. Hence it is β' active, decaying to the target
nucleus:
\[ \frac{A}{Z} Y \rightarrow \frac{A}{Z-1} X \]
Since \( Q(β') = M_Y - M_X \), the Q of the (n, p) reaction is
\[ Q(n, p) = B_Y - B_X = M_x + M_R - M_Y - M_R \]
\[ = (M_r - M_R) - Q(β') \]
\[ = 0.782 - Q(β') \text{ MeV} \]
So if \( Q(β') < 0.782 \text{ MeV} \), the reaction is exoergic. For
\( Q(β') > 0.782 \text{ MeV} \), the reaction is endoergic. Examples of some (n, p)
reactions are given below:
\[ \frac{3}{2} \text{He} + \frac{1}{2} n \rightarrow \frac{4}{2} \text{He}^* \rightarrow \frac{1}{2} \text{H} + \frac{1}{2} \text{H} \quad (Q = 0.764 \text{ MeV}) \]
\[ \frac{14}{7} \text{N} + \frac{1}{2} n \rightarrow \frac{15}{7} \text{N}^* \rightarrow \frac{14}{6} \text{C} + \frac{1}{2} \text{H} \quad (Q = 0.627 \text{ MeV}) \]
\[ \frac{27}{13} \text{Al} + \frac{1}{2} n \rightarrow \frac{28}{13} \text{Al}^* \rightarrow \frac{27}{12} \text{Mg} + \frac{1}{2} \text{H} \quad (Q = -1.83 \text{ MeV}) \]

Nuclear Reactions (1)

Libby's method of estimating the ages of archeological and
anthropological samples:

Of the above reactions, the second is of particular importance. The
product nucleus \( ^{14}\text{C} \) is β active with a half-life \( \tau = 5568 \) y. The maximum
energy is 0.155 MeV. The \( ^{14}\text{C} \) isotope is continually produced by \( n, p \)
reaction in the atmosphere due to the absorption of the cosmic ray
neutrons by the \( ^{14}\text{N} \) nuclei in air. The American physicist W.F. Libby and
his co-workers have developed a method of measuring the ages of
anthropological and archeological samples by estimating the amounts of
\( ^{14}\text{C} \) present in them.

All living systems, including plants and trees, ingests the radio-carbon
\( (^{12}\text{C}) \) by exchange processes and its concentration in a living system
reaches equilibrium value. After the death of the system, the exchange
stops and the amount of radio-carbon decreases due to β decay with the
half-life given above. Libby and his co-workers have developed a special
counter, based on anti-coincidence counting technique, to detect the
extremely weak β-activity in an old archeological (or anthropological)
sample. Comparing this with a similar recently acquired sample (living or
just dead), it is possible to estimate the age of the old sample. By this
method it is possible to measure the ages of samples several thousand
years old.

(n, d) and (n, t) reactions:

These reactions are known as pick-up-reactions which are just the
opposite of stripping reactions mentioned earlier and belong to the
category of direct reactions. The mechanism of these reactions is different
from compound nuclear process. An example of the \( n, t \) type of pick up
reaction is
\[ ^{14}\text{N} + \frac{1}{2} n \rightarrow ^{12}\text{C} + \frac{1}{2} \text{H} \]
It is believed that traces of tritium \( (^3\text{H}) \) found in nature in the form of
\( ^3\text{H}_2\text{O} \) mixed with ordinary water is due to the formation of \( ^3\text{H} \) in the
atmosphere by the interaction between cosmic ray neutrons and the
nitrogen nuclei in the atmospheric air to produce the above reaction.

(n, γ) reaction:
The most important neutron-induced reaction is the \( n, γ \) reaction,
known as the radiative capture of neutrons. The general formula for such
reaction is
\[ \frac{A}{Z} X + \frac{1}{2} n \rightarrow \frac{A+1}{Z} C^* \rightarrow \frac{A+1}{Z} Y + γ \]
Here the product nucleus is the same as the compound nucleus in the
ground state \( (Y = C) \). The \( (n, γ) \) reactions are always exoergic \( (Q > 0) \) and
can be induced by almost zero energy neutrons. The \( Q \) of the reaction is
\[ Q(n, γ) = M_X + M_n - M_Y \]
Except for some light nuclei \( Q(n, γ) \approx 8 \text{ MeV} \). Thus in the \( (n, γ) \)
reaction induced by zero energy neutrons, γ-rays with energies up to about
8 MeV are emitted.
In 1934 a group of scientists E. Fermi, E. Amaldi, O. D’Agostino, F. Rasetti and E. Serge made the very important discovery in the University of Rome that the probability of occurrence of the \((n, \gamma)\) reaction was greatly enhanced if the reaction was induced by very slow neutrons. They placed a Ra-Be neutron source in a large tank filled with water or inside a block of paraffin. The fast neutrons coming out of the source suffered repeated collisions with the protons in the hydrogen atoms within the medium (water or paraffin). Because of their equal masses, the neutrons lost a large fraction of their energy at each elastic impact with the protons (see § 10.4). Their energy was thus reduced to the kinetic energy of random thermal motion \((3/2)kT\) of the molecules in the medium after a few collisions. Such neutrons are known as thermal neutrons. At ordinary room temperature \((T \sim 300\ \text{K})\), the energy is about \((\sim 0.026\ \text{MeV})\).

Fermi and his associates exposed a large number of elements in the periodic table to the action of these thermal neutrons and observed that in many cases they gave rise to the production of radioactive product nuclei. By comparing with the radioactivity induced in the same elements by fast neutrons coming out of the source, they concluded that the discovery of the \((n, \gamma)\) reaction was greatly enhanced by the use of thermal neutrons. In some cases, the increase was by a factor 1000 or more. Their discovery led to the development of the method of large scale production of artificially radioactive substances.

Examples of some \((n, \gamma)\) reactions are:

\[
\begin{align*}
\text{H} + \text{n} &\rightarrow \text{H} + \gamma \\
\text{H} + \text{n} &\rightarrow \text{H} + \gamma \\
\text{Na} + \text{n} &\rightarrow \text{Na} + \gamma \\
\text{Cu} + \text{n} &\rightarrow \text{Cu} + \gamma \\
\text{Rh} + \text{n} &\rightarrow \text{Rh} + \gamma \\
\text{Ag} + \text{n} &\rightarrow \text{Ag} + \gamma \\
\text{In} + \text{n} &\rightarrow \text{In} + \gamma \\
\text{Au} + \text{n} &\rightarrow \text{Au} + \gamma
\end{align*}
\]

Except in the first case, the product nuclei in all the above reactions are radioactive. Capture of a neutron by the target nucleus increases the neutron-proton ratio and hence shifts the nucleus to the left above the stability line (see Ch. IX). Hence the product nucleus usually becomes \(\beta^-\) active, since it has an excess of neutrons compared to the number of protons which would make it stable. However in the case of some odd-odd product nuclei (e.g., \(^{64}\text{Cu}\) and \(^{108}\text{Ag}\) ) both \(\beta^-\) and \(\beta^+\) (or electron-capturing) activities are observed (see Ch. IX). The radioactive decay schemes in the above three cases are given on next page:

**Nuclear Reactions (1)**

\[
\begin{align*}
^{29}\text{Cu} &\rightarrow ^{30}\text{Zn} & \tau = 12.8\ \text{h} \\
\beta^- &\rightarrow ^{64}\text{Ni} \\
^{108}\text{Ag} &\rightarrow ^{108}\text{Cd} & \tau = 2.3\ \text{min} \\
\beta^+ &\rightarrow ^{108}\text{Pd} \\
\end{align*}
\]

In some cases, the radiative capture of neutrons shows exceptionally high probability at certain definite neutron kinetic energy. This is known as the resonant capture of neutrons. Thus the reaction \(^{115}\text{In}(n, \gamma)^{116}\text{In}\) shows a peak in the cross section curve at \(E_n = 1.44\ \text{MeV}\) at which the cross section is \(3 \times 10^3\) barns (see Fig. 11.1.3 in Ch. XI).

More than one particle emission:

At high neutron energies \((E_n > 8\ \text{MeV})\), more than one particle may be emitted from the compound nucleus formed by the capture of a neutron. The corresponding reactions are \((n, 2n), (n, 3n), (n, p\gamma), (n, 2\gamma)\) etc. For the proton emission along with one or two neutrons, the energy of excitation of the compound nucleus must be sufficiently high so that the proton is able to cross the potential barrier to come out.

10.16 Gamma-ray induced reactions

These reactions, also known as the photo-nuclear reactions, occur if sufficiently high energy photons enter into a nucleus. The energy of the incident \(\gamma\)-ray must be greater than the energy of binding of a nuclear particle (separation energy \(S_p\) ) e.g., a neutron, a proton, an \(\alpha\)-particle etc. for the particle to be emitted from the nucleus producing reactions of the type \((\gamma, n), (\gamma, p), (\gamma, \alpha)\) etc. These reactions are endoergic.

An example of the \((\gamma, n)\) reaction is:

\[
\text{H} + \gamma \rightarrow \text{H} + n
\]

This is known as the photo-disintegration of the deuteron. The energy of the \(\gamma\)-ray which can induce this reaction must be greater than the binding energy \((2.226\ \text{MeV})\) of the deuteron. In fact this reaction has been used in the measurement of the deuteron binding energy accurately. In these measurements \(\gamma\)-rays from the naturally radioactive isotope Th\(^{229}\) with an energy of \(2.62\ \text{MeV}\) and those from the artificially radioactive \(^{24}\text{Na}\) isotope \((E_\gamma = 2.76\ \text{MeV})\) were used. Conservation laws for energy and momentum were used to determine \(B_\gamma\).

Another example of \((\gamma, n)\) reaction is:

\[
\text{Be} + \gamma \rightarrow \text{Be}^* \rightarrow \text{Be} + n
\]

The threshold of this reaction is \(1.66\ \text{MeV}\). This reaction is used for the preparation of photo-neutrons source (see Ch. XIII). \(\gamma\)-rays from \(^{24}\text{Na}\) or more preferably from the isotope \(^{124}\text{Sb}\) of antimony \((Z = 51)\) is
used for this purpose because of the relatively longer half-life of the latter ($\tau = 60$ days). The maximum $\gamma$-energy from this source is $E_{\gamma} = 2.04$ MeV.

Some examples of different types of photo-nuclear reactions are:

$^8$Be($\gamma$, $p$)$^8$Li ; $^{25}$Mg($\gamma$, $p$)$^{24}$Na ; $^{10}$B($\gamma$, $d$)$^8$Be ; $^{12}$C($\gamma$, $\alpha$)$^8$Be

At high $\gamma$-energies, multiparticle emission from the excited nucleus $X^*$ may take place. Reactions of the type ($\gamma$, 2n), ($\gamma$, pn), ($\gamma$, 2p) etc. have been observed.

It may be noted that the deuteron and $^8$Be are the only nuclei which undergo photo-disintegration by $\gamma$-rays from naturally radioactive substances. For all other nuclei, $\gamma$-rays from nuclear reactions (mainly proton-induced) of much higher energies ($E_{\gamma} > 2.62$ MeV) must be used to produce photo-nuclear reactions. Some of these are :

$^7$Li($p$, $\gamma$)$^8$Be($E_{\gamma}$ = 17.2 MeV) ; $^{11}$B($p$, $\gamma$)$^{12}$C($E_{\gamma}$ = 11.7 MeV) ;

$^{19}$F($p$, $\alpha$)$^{16}$O* , $^{16}$O* $\rightarrow$ $^{16}$O + $\gamma$($E_{\gamma}$ = 6.13 MeV).

The mechanism of photo-nuclear reactions will be discussed in § 11.17.

10.17 Special types of nuclear reactions

Apart from the reactions discussed above, which mostly take place through compound nucleus formation, various special types of reactions are known for which the reaction mechanisms are different. These include the already mentioned direct reactions, photo-nuclear reactions, heavy-ion reactions, nuclear fission, Coulomb excitation etc.

Some of these will be discussed later (see Ch. XI).

References


Problems

1. A beam of 7.3 MeV $\alpha$-particles is used to produce the reaction $^{27}$Al($\alpha$, $p$)$^{30}$Si. The protons emitted at 0° are found to have an energy of 9.34 MeV. What is the $Q$ value of the reaction? (Hint: Use Eq. 10.5-7) Check the result using the mass values. ($2.19$ MeV)

2. The $Q$ values of the reactions $^2$H ($d$, $t$) $^1$H and $^2$H ($d$, $^3$He) $^1$H are 4.032 and 3.269 MeV respectively. The $\beta$-disintegration energy of $^2$H is known to be 0.019 MeV. Calculate the $\beta$-disintegration energy of the neutron from the above data. ($0.783$ MeV)

3. Deuterons of 1.51 MeV energy are used to produce the reaction $^6$O ($d$, $\alpha$)$^{14}$N. The $\alpha$-particles emitted at 90° are found to have the energy 3.427 MeV. Calculate the $Q$-value of the reaction and check the same from the masses of the nuclei. ($3.112$ MeV)

4. A 60 $\mu$A beam of 6 MeV protons accelerated in a cyclotron falls on an 18 mg/cm$^2$ thick copper foil of 1 cm$^2$ area normally for 5 minutes to produce the reaction $^{63}$Cu ($p$, $n$)$^{63}$Zn. If the reaction cross section is 0.093 barn and $^{63}$Zn has a half-life of 38 min, what is the number of $^{63}$Zn nuclei produced immediately after the bombardment? Find the rate of disintegration of these atoms. ($1.18 \times 10^{12} ; 3.59 \times 10^8$ s$^{-1}$).

5. A beam of 1.2 MeV protons from a Cockcroft-Walton generator bombard a tritium ($^3$H) target to produce ($p$, $n$) reaction. What will be the kinetic energy of the neutrons emitted at 90° w.r.t. to the incident beam? What is the threshold energy of the reaction? (Hint: Use Eqs. 10.5-10 and 10.5-13). ($0.409$ MeV)

6. A nucleus of mass $M$ absorbs a photon of energy $h\nu$. Show that the energy of excitation of the nucleus is given by

$$M^2 \left[ 1 + \frac{2h\nu}{M^2} \right]^{1/2} - 1$$