Nuclear Fission and Nuclear Fusion

14.1 Discovery of nuclear fission

Nuclear fission is a special type of nuclear reaction in which an excited compound nucleus breaks up generally into two fragments of comparable mass numbers and atomic numbers. Fission usually occurs amongst the isotopes of the heaviest elements known, e.g., uranium, thorium etc.

Nuclear fission was discovered by the two German chemists Otto Hahn and F. Strassmann in 1939. It happened to be one of the most important discoveries in nuclear physics, since it paved the way for the utilization of the internal energy of the nucleus for practical purposes.

We have seen earlier that Enrico Fermi and his associates in Rome investigated the neutron capture (n, γ) reactions by various nuclei using neutrons, slowed down to very low energies (1934). Along with other elements, when they bombarded uranium (Z = 92), the last naturally occurring element in the periodic table by slow neutrons they found evidence for the production of a few β⁻ active isotopes of short half-lives. One of these was the 23 min ²³⁹U isotope, which by β⁻ emission should transform into the isotope ²³⁹Np of the first transuranic element neptunium (Z = 93). Though this transformation was later actually observed from the radioactivity of ²³⁹Np, Fermi and his associates failed to establish definitely that such transformation had actually taken place.

Subsequently, Hahn and Strassmann, along with Lise Meitner in Germany and independently I. Curie-Joliot and L. Savich in France, tried to identify chemically the radioactive products produced by the neutron bombardment on uranium. The former group observed that one of the products was chemically similar to the element barium (Z = 56), while the latter group found evidence that one of the products was similar to the element lanthanum (Z = 57).

Since the mass number and atomic number of barium or lanthanum are much smaller than those of uranium, it was not possible to explain the above observations on the basis of the general ideas about nuclear reactions prevalent in those days. It was believed that during a nuclear reaction, generally a nucleon (e.g., a proton or a neutron) or a group of them comprising a light nucleus like a deuteron or an α-particle, was emitted. So the residual nucleus produced in a reaction could differ from the target nucleus by only a few units in A and Z. So it was at first thought that the barium-like element, identified in the experiments of Hahn and his associates, was in reality an element heavier than barium and chemically similar to the latter. A glance at the periodic table shows that the element radium with Z = 88 falls in the same vertical column as barium (Group II-B). Hence they are chemically similar. An isotope of radium could conceivably be produced from uranium bombarded with neutrons by the emission of two α-particles.

Soon afterwards, Hahn and Strassmann, by very careful chemical analysis, definitely established that it was barium, and not radium, that was produced as one of the reaction-products, when uranium was bombarded with slow neutrons. They also found lanthanum and cerium (Z = 58) amongst the reaction-products. Subsequently, Meitner and her nephew Otto Frisch, working in Sweden (to which country they had escaped to save themselves from Nazi persecution in Germany) provided the correct explanation of Hahn and Strassmann’s results by suggesting that the uranium nucleus bombarded with neutrons broke up into two large fragments. They gave the name nuclear fission to this new phenomenon.

Since the atomic number of barium is 56, the other fragment produced in the fission of uranium should have the atomic number (92 – 56) or 36. It should thus be the nucleus of an isotope of krypton, which was subsequently identified. The two fragments produced in fission are known as fission fragments.

The nuclear fission discovered by Hahn and Strassmann can be symbolically written as

\[ ^{235}_{92} U + ^{1}n \rightarrow ^{236}_{92} U^{*} \rightarrow ^{56}_{56} Ba + ^{36}_{36} Kr \]  

...(14.1-1)

Because of the uncertainties in assigning the mass numbers to the fission fragments, these are not shown in the above equation.

Since the fission fragments are heavy and carry positive charges, which are many times the electronic charge (high Z) they are expected to produce intense ionization in gases. This was confirmed by Frisch, who observed large electrical pulses produced by them in an ionization chamber connected to a linear amplifier.

In Fig. 14.1 is shown the cloud chamber photograph of the tracks of the two fission fragments, travelling in opposite directions from a vertical thin foil containing uranium. The almost horizontal tracks of the fragments can be recognized by the little lateral branches caused by collisions with the light nuclei in the chamber gas (see Eq. 10.4-27 in Ch. X). The nature of the tracks confirm that they were produced by heavy ions of comparable masses.

The discovery of nuclear fission was announced in the early part of 1939. Soon afterwards, it was confirmed by experiments in many
be heated due to the absorption of the fission fragments and some of the other products. The heat thus generated can be measured by calorimetric method, which gives a value of about 186 MeV per uranium nucleus fissioned. This falls short of the value given in the above table. This is due to the fact that the antineutrinos and \( \gamma \)-rays produced have very high penetrability and hence escape from the uranium piece.

The enormous quantity of energy release in nuclear fission can be understood qualitatively with the help of the binding fraction \( f_b \) curve given in Ch. II (see Fig. 2.2). A heavy nucleus like uranium has a value of \( f_b = B/A = 7.6 \) MeV per nucleon. The fragments produced in its fission have mass numbers near the middle of the periodic table and hence the values of \( f_b \) for them are 8.5 MeV per nucleon. Thus, during the fission process, about 0.9 MeV energy per nucleon is released, so that the total energy release is around \( 238 \times 0.9 = 212 \text{ MeV} \).

We can determine the energy release in fission quantitatively from the known atomic masses of the nuclei involved. If we assume that three prompt neutrons are released when the fission occurs, we can write in a typical case

\[
\frac{235}{92} \text{U} + \frac{1}{0} n \rightarrow \frac{236}{92} \text{U}^* \rightarrow \frac{141}{56} \text{Ba} + \frac{92}{36} \text{Kr} + \frac{3}{0} n
\]

From the mass energy equivalence principle, we then get

\[
Q = M (\frac{235}{92} \text{U}) + M (\frac{141}{56} \text{Ba}) - M (\frac{92}{36} \text{Kr}) - 3M_n
\]

\[
= 235.04278 + 1.00866 - 140.9129 - 91.89719 - 3 \times 1 \times 1.00866
\]

\[
= 0.21537 \; u = 200.6 \text{ MeV}
\]

Energies of more or less the same order of magnitude are released in the fission of other nuclei.

Because of this enormous energy release during fission, it is possible to obtain very large quantity of energy by the nuclear fission of a small amount of uranium. For example if 1 g of \( ^{235} \text{U} \) is completely fissioned, we can calculate the energy released from the above estimate of the \( Q \) value:

The number of atoms of \( ^{235} \text{U} \) per kilogram is

\[
n = \frac{6.025 \times 10^{23} \times 10^3}{235} = 2.564 \times 10^{24}.
\]

Hence the energy release per gram of \( ^{235} \text{U} \) is

\[
E = \frac{nQ}{10^3} = \frac{2.564 \times 10^{24}}{10^3} \times 200.6 \times 1.6 \times 10^{-13}
\]

\[
= 8.229 \times 10^9
\]

\[
= 2.29 \times 10^9 \text{ kWh}
\]

A thermal power generator having a capacity of 1 MW (heat) would have to be run for 229 hours to generate this amount of energy. The mass of coal which must be burnt to produce an equivalent amount of energy can be estimated as follows. Since the energy release in the chemical

\[\text{Table 14.1}\]

<table>
<thead>
<tr>
<th>Components</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic energy of the fission fragments</td>
<td>167</td>
</tr>
<tr>
<td>Kinetic energy of the prompt neutrons</td>
<td>5</td>
</tr>
<tr>
<td>Energy of the prompt ( \gamma )-rays</td>
<td>6</td>
</tr>
<tr>
<td>Energy of the ( \beta^- )-particles emitted by the fission fragments</td>
<td>8</td>
</tr>
<tr>
<td>Energy of the antineutrinos emitted by the fission fragments</td>
<td>12</td>
</tr>
<tr>
<td>Energy of the ( \gamma )-rays emitted by the fission fragments</td>
<td>6</td>
</tr>
<tr>
<td>Total energy release in fission</td>
<td>204</td>
</tr>
</tbody>
</table>
process of burning of coal ($C + O_2 = CO_2$) is 4 eV per atom of carbon, the quantity of energy released when 1 kg of carbon is completely burned 

$$e = 4 \times 1.6 \times 10^{-19} \times 6.025 \times 10^{23} \times 10^3$$

$$e = 3.213 \times 10^7 \text{ J}$$

$$e = 8.926 \text{ kWh}$$

So mass of carbon required is

$$m = \frac{2.29 \times 10^4}{8.926} = 2.56 \times 10^3 \text{ kg}$$

The above estimates clearly demonstrate the advantage of using uranium for power generation.

### 14.3 Nature of the fission fragments

We have seen that in the nuclear fission of a heavy element like uranium, two fission fragments of comparable mass numbers are produced. Actually the mass numbers of the fragments are not the same in most cases. There is a certain spread in the distribution of the mass numbers, as shown in Fig. 14.2. The mass numbers of the two fragments are generally different. In the case of the thermal neutron fission of $^{235}\text{U}$, the lighter fragments have the mass number spread between 85 to 105 with a broad peak at $A \approx 96$. The heavier fragments have mass numbers distributed between 130 to 150 with a broad peak at $A \approx 138$. Thus the fission is asymmetric in this case. The curve shows that the probability of symmetric fission with equal mass numbers for the fragments is the least.

If the excitation energy of the compound nucleus undergoing fission is increased (higher neutron energy) symmetric fission becomes more probable. Symmetric fission is also more probable when fission is induced by particles other than neutrons.

The fission yield curve shown in Fig. 14.2 is symmetrical about the central minimum, since for every heavy fragment there is a corresponding light fragment with the same percentage yield.

The reason for asymmetry in the fission fragment mass distribution is not fully understood. It seems to be related to the shell structure of the nucleus (see §14.10).

The fission fragments are highly neutron rich. The two fission fragments $^{141}\text{Ba}$ and $^{92}\text{Kr}$ in Eq. (14.2-1) have the neutron excesses $N-Z = 13$ and 22 respectively. Natural barium and krypton have 7 and 6 stable isotopes respectively. Of these, the heaviest are $^{138}\text{Ba}$ with $N-Z = 26$ and $^{86}\text{Kr}$ with $N-Z = 14$ respectively. In other words, the above two fission fragments have neutron excesses considerably greater than are needed for their stability. The nuclear stability condition against decay discussed in Ch. IX (Fig. 9.1) tells us that these nuclei cannot be stable; they must be $\beta^+$ active.

The $\beta^+$ activities of the above two fission fragments can be represented by the following $\beta^+$-chains:

\[
\begin{align*}
141\text{Ba} & \rightarrow 141\text{La} & 141\text{Ce} & \rightarrow 141\text{Pr} \text{ (stable)} \\
141\text{La} & \rightarrow 141\text{Ce} & 141\text{Pr} & \text{ (stable)} \\
92\text{Kr} & \rightarrow 92\text{Rb} & 92\text{Sr} & \rightarrow 92\text{Y} & 92\text{Zr} \text{ (stable)}
\end{align*}
\]

The reason for high neutron excesses in the fission fragments can be easily understood.

We have seen that the neutron excess ($N - Z$) increases with the mass number $A$. For the heaviest element uranium, two stable isotopes $^{235}\text{U}$ and $^{238}\text{U}$ have $(N - Z)$ values 51 and 54 respectively. On the other hand, the sum of the neutron excesses for the heaviest stable isotopes of Ba and Kr (which are in the middle region of the periodic table) given above is 40. Thus when a uranium nucleus undergoes fission, both the fission fragments generally contain many neutrons in excess of the numbers required for their stability, which makes them $\beta^+$ active. Even if one of the fragments has the proper neutron excess to make it stable, the other must have so great a neutron excess that it will definitely be $\beta^+$ active.

### 14.4 Energy distribution between the fission fragments

The asymmetry in the mass distribution of the fission fragments discussed above is reflected in the distribution of the kinetic energy between them.

The kinetic energies of the fission fragments have been measured using ionization chamber within which a thin foil of U or Th is placed.
The ions produced by the two fragments that appear simultaneously in the two halves of the chamber are collected in an electrostatic field and the ionization currents measured by two electrometers calibrated by α-particles of known energy. It is generally assumed that the energy required to produce an ion pair in argon is the same both for the fission fragments as well as for the α-particles and the total energy is proportional to the total ionization current.

Neglecting the masses of the prompt neutrons emitted in fission, we have from the conservation of momentum for the two fission fragments of masses \( M_1 \) and \( M_2 \)

\[
M_1v_1 = M_2v_2
\]

Hence the ratio of the energies is

\[
\frac{E_1}{E_2} = \frac{M_2v_2^2}{M_1v_1^2} = \frac{M_2}{M_1}
\]

It is thus possible to determine the total energy \( (E_1 + E_2) \) for each fission, as also the ratios \( E_1/E_2 \) and \( M_1/M_2 \).

It may be noted that the mass distribution curve discussed in the previous section (Fig. 14.2) was determined by this method.

The energy distribution between the two fission fragments in the case of the thermal neutron fission of \(^{235}\)U is shown in Fig. 14.3. There are two broad peaks. The higher energy peak goes with the lighter fragments and vice-versa. The lower energy peak is broader than the other, such that the areas under them are equal when properly corrected for the overlapping region.

![Fig. 14.3. Energy distribution of the fission fragments.](image)

### 14.5 Emission of neutrons in nuclear fission

The fission fragments, which are produced by the breakup of the compound nucleus (e.g., \(^{236}\)U* in Eq. 14.2-1) should be such that the mass numbers add up to the mass number of the fissioning nucleus (236). They may be called the primary fragments. Because of the very large neutron excesses in them, one or both of them, generally emit a few neutrons to reduce neutron excess to some extent. These are the fragments that are finally left behind, which by successive β-decay ultimately end up in the two stable products, as discussed in § 14.2. The emission of the neutrons from the primary fragments mostly occurs within about 10⁻¹³ s or their formation. These are known as prompt neutrons. There are reasons to believe that about 10% of them may be emitted within 10⁻¹⁰ s.

For \(^{235}\)U undergoing fission by thermal neutrons, about 2.5 neutrons are emitted on the average per fission. For the thermal neutron fission of \(^{239}\)Pu, this number is about 3 per fission. These neutrons are emitted with a distribution of energy ranging from 0.05 MeV to 17 MeV; i.e., they are emitted as fast neutrons. Their energy distribution is approximately Maxwellian with an average of about 2 MeV (see Fig. 14.4). Besides the prompt neutrons, in a small fraction of cases the neutrons are emitted with half-lives of the order of seconds from the fission fragments. These are known as delayed neutrons.

About 0.75% of the neutrons are emitted as delayed neutrons. Their emission follows the exponential law, as in the case of radioactive decay. In the case \(^{235}\)U + \(^1\)H fission, delayed neutron emission half-lives from 0.05 s to 56 s have been observed. In the last case, delayed neutron emission is observed from the \(^{87}\)Br (Z = 35) isotope produced as a fission fragment. Actually this is β-active:

\[
{^{87}\text{Br}} \rightarrow ^{87}\text{Kr}^* (\tau = 56 \text{ s})
\]

The decay product \(^{87}\text{Kr}^*\) which is formed in an excited state, emits a neutron as soon as it is formed:

\[
^{87}\text{Kr}^* \rightarrow ^{86}\text{Kr} + ^1\text{H}
\]

It may be noted that the neutron number in \(^{87}\text{Kr}\) is \( N = 51 \) which is reduced to the magic number \( N = 50 \) in \(^{86}\text{Kr}\) formed by neutron emission from the former. Because the outermost neutron shell is completely filled up in \(^{86}\text{Kr}\), the last (i.e., the 51st) neutron in \(^{87}\text{Kr}^*\) is so loosely bound that it is emitted as soon as it is formed by the β-decay of \(^{87}\text{Br}\). The energy level diagram in this decay is shown in Fig. 14.5.

The emission of the prompt neutrons in nuclear fission makes possible the realization of nuclear chain reaction, which is the fundamental basis for the operation of nuclear reactors (see Ch. XV).
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where \( f_b' \)’s are the binding fractions.

For \( Q_f \) to be positive, \( \Delta f_b \) must be positive which happens if \( f_{b'} > f_b \), i.e., the binding fraction of the product nuclei is greater than that of the parent nucleus. A glance at the binding fraction curve in Fig. 2.2 shows this to be the case for the nuclei in the descending part of the curve beyond the maximum. In fact it was from this consideration that we obtained in § 14.2, a rough estimate of the energy released in fission in the case of a heavy nucleus (uranium).

Writing the atomic masses in terms of the semi-empirical mass formula derived in Ch. IX, we have, neglecting the pairing energy term

\[
\begin{align*}
M(A, Z) &= M_M + NH_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \left( A - 2Z \right)^2 \\
M\left( \frac{A}{2}, \frac{Z}{2} \right) &= \frac{Z^2}{2} M_M + \frac{N}{2} M_n - a_1 A + a_2 A^{2/3} + a_3 \frac{(Z/2)^2}{A^{1/3}} + a_4 \left( A - 2Z \right)^2 \\
\text{Hence } Q_f &= M(A, Z) - 2M \left( \frac{A}{2}, \frac{Z}{2} \right) \\
&= a_2 A^{2/3} \left( 1 - \frac{2}{2^{2/3}} \right) + a_3 \frac{Z^2}{A^{1/3}} \left( 1 - \frac{2^{1/3}}{2} \right) \\
&= -0.26 a_2 A + 0.37 a_3 \frac{Z^2}{A^{1/3}} \quad \ldots (14.6-2)
\end{align*}
\]

Thus the symmetric S.F. will be energetically possible (\( Q_f > 0 \)) if

\[
\frac{Z^2}{A} \geq \frac{0.26 a_2}{0.37 a_3}
\]

Substituting the values \( a_2 = 0.019114 \) u and \( a_3 = 0.0007626 \) u, we get

\[
\frac{Z^2}{A} \geq 17.6
\]

This condition is found to be fulfilled for \( A > 90 \) and \( Z > 40 \). (For \( A = 90, Z = 40, Z^2/A = 17.8 \). Thus for nuclei for which \( A > 90 \), S.F. should be energetically possible. In reality however, it is a very uncommon phenomenon. Even amongst the nuclei of the heaviest atoms in the periodic table, e.g., uranium, it is very rarely observed. For instance there is only about one S.F. per hour in 1 g of \( ^{235}\text{U} \) corresponding to a half-life of \( 2 \times 10^{17} \) yr.

The reason for this lies in the quantum mechanical barrier penetration problem, which we discussed in connection with the \( \alpha \)-disintegration of nuclei. The problem is much more acute in the present case, since the nuclei of the fission fragments carry much higher charges than the \( \alpha \)-particles.

Let us consider the reverse case in which two spherical fission fragment nuclei of mass number \( A/2 \) and carrying positive charge \( Ze/2 \) each are brought towards each other from infinity. \( A \) and \( Z \) are the mass number and atomic number of the parent nucleus the symmetric spontaneous fission of which produces the above two. At infinity, their mutual
potential energy is zero while at a distance $r$ between their centres, the electrostatic potential energy rises to

$$V_e = \frac{1}{4 \pi \varepsilon_0} \frac{(Ze/2)^2}{r} \quad \cdots(14.6-3)$$

This potential energy is positive since the force is repulsive. As $r$ decreases, $V_e$ increases as shown in Fig. 14.6a. It should be maximum at B when the two fragments just touch each other which happens when their centres are separated by $2R'$ where $R' = r_0 (A/2)^{1/3}$ is the radius of each fragment. However, actually the maximum is reached when the fragments begin to coalesce to produce the original nucleus under the action of strong short range attractive interaction (nuclear force). The highest point on the actual potential energy curve $Q$ is below B. From this point where the potential is $E_b$, the course of the potential energy curve towards $r = 0$ is not known exactly.

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The point P giving the energy at $r = 0$ corresponds to the mass energy of the parent nucleus. In order that the parent nucleus may undergo spontaneous fission, the two fragments must cross the potential barrier at the highest point $Q$.

The heights of the points P, P', P'' etc. in Fig. 14.6b for different A values above the zero energy line (complete separation of the fission fragments) correspond to the $Q_f$ values calculated above (Eq. 14.6-2). As long as these points are above the zero energy line, spontaneous fission is energetically possible. This is the case for nuclei with $A > 90$. However, because of the barrier penetration problem, there is very little probability of the fission to take place, since the Gamow factor discussed in Ch. IV is extremely small.

In Fig. 14.7, the barrier height $E_b$ calculated from Bohr–Wheeler theory (a) and the values of $Q_f$ (b) are plotted as functions of A for comparison. It will be seen that $E_b > Q_f$ for the nuclei with $A \leq 250$. $E_f = E_b - Q_f$ is called the activation energy for fission. So for these nuclei, the activation energy is positive. S.F. is possible in this case through barrier penetration only. This is the reason why S.F. is so rare a phenomenon. In the potential energy diagrams of Fig. 14.6b these correspond to the cases where the humps of the potential energy curves are above the rest energies of the parent nuclei.

Fission can be induced in these nuclei if energy is supplied to them by particle (neutron) or radiation ($\gamma$-ray) absorption. If the neutron separation energy from the target nucleus is $S_n$ and the neutron kinetic energy is $E_n$, then the energy of excitation of the latter is

$$E_e = E_n + S_n \quad \cdots(14.6-4)$$

For fission to occur, this must be greater than the deficiency of $Q_f$ below $E_b$, i.e.,

$$E_e \geq E_b - Q_f \quad \cdots(14.6-5)$$

The minimum energy required for the fission to occur is thus

$$E_{e\min} = E_b - Q_f = E_f \quad \cdots(14.6-6)$$

Bohr and Wheeler, on the basis of the liquid drop model of the nucleus, developed the theory for the calculation of the activation energy $E_f$. This will be discussed in § 14.7. In Table 14.2 are listed the values of
$E_f$ as also the values of $S_n$ and the minimum kinetic energy $E_{min} = E_f - S_n$ required to produce fission in a few heavy nuclei.

The energy $E_{min} = E_f - S_n$ is sometimes called the fission threshold energy. But fission being exoergic, cannot have any threshold in the real sense. However, since spontaneous fission by barrier penetration is a very rare phenomenon, for all practical purposes, S.F. requires a minimum neutron energy $E_{min}$ to take place, which justifies in a way, the term fission threshold for it. While $^{232}\text{Th}$ and $^{238}\text{U}$ have positive fission thresholds, $^{235}\text{U}$, $^{237}\text{U}$ and $^{239}\text{Pu}$ have negative fission thresholds.

Table 14.2

<table>
<thead>
<tr>
<th>Target nucleus</th>
<th>Compound nucleus</th>
<th>$S_n$ (MeV)</th>
<th>$E_f$ (MeV)</th>
<th>$E_{min}$ (MeV)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{232}\text{Th}$</td>
<td>$^{233}\text{Th}$</td>
<td>5.0</td>
<td>6.6</td>
<td>1.6</td>
<td>Only fast fission is possible</td>
</tr>
<tr>
<td>$^{233}\text{U}$</td>
<td>$^{234}\text{U}$</td>
<td>7.5</td>
<td>4.6</td>
<td>0</td>
<td>Thermal neutron fission possible</td>
</tr>
<tr>
<td>$^{235}\text{U}$</td>
<td>$^{236}\text{U}$</td>
<td>6.5</td>
<td>5.3</td>
<td>0</td>
<td>Only fast fission possible</td>
</tr>
<tr>
<td>$^{237}\text{Np}$</td>
<td>$^{238}\text{Np}$</td>
<td>5.4</td>
<td>6.2</td>
<td>0.8</td>
<td>Only fast fission possible</td>
</tr>
<tr>
<td>$^{239}\text{U}$</td>
<td>$^{240}\text{Pu}$</td>
<td>4.8</td>
<td>6.3</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>$^{239}\text{Pu}$</td>
<td>$^{240}\text{Pu}$</td>
<td>6.5</td>
<td>4.0</td>
<td>0</td>
<td>Thermal fission possible</td>
</tr>
</tbody>
</table>

For $A > 250$, $Q_f > E_b$ (see Fig. 14.7) there is no longer any hump in the potential energy curve, so that the parent nucleus can undergo spontaneous fission, without the fragments having to cross any potential barrier. So these nuclei cannot exist in nature, since they will undergo instantaneous spontaneous fission if they are somehow formed. In Fig. 14.6 the potential energy diagram for $A = 280$ corresponds to such a case. The points $P'$ (rest mass of the parent nucleus) and $Q'$ ($E_b$) are on the same horizontal line in this case.

In Fig. 14.6a, we have marked the point S on the Coulomb part of the potential energy diagram, which is known as the scission point. This corresponds to the two fission fragments just separated from each other by crossing over the barrier in the case of neutron induced fission. They then fly apart and gain their observed kinetic energies when they are infinite distance apart. The time scale involved in the evolution of the nucleus from the highest point of the barrier $Q$ to the scission point $S$ is $\sim 10^{-20}$ s. It is believed that about 10% of the prompt neutrons are emitted during this interval or just at the scission point.

Table 14.2 shows that the even-even nuclei $^{232}\text{Th}$ ($Z = 90$) and $^{238}\text{U}$ ($Z = 92$) can undergo fission only with fast neutrons. The even-odd nuclei $^{235}\text{U}$, $^{233}\text{U}$ and $^{239}\text{Pu}$ ($Z = 94$) however, can undergo thermal neutron fission. The reasons for these can be understood if we refer to the Bethe-Weizsäcker semi-empirical mass formula. This formula contains a pairing energy term $\delta_e = 0.036A^{-3/4}$ u which is positive ($> 0$) for odd-odd, negative ($< 0$) for even-even, zero for even-odd or odd-even nuclei. We can write the equation for the fission of a nucleus $^{232}\text{X}$ induced by neutron absorption as

$$^{232}\text{X} + \frac{1}{0}n \rightarrow ^{A+1}\text{C*} \rightarrow \text{Fission}$$

$\text{C*}$ is the excited compound nucleus. Its excitation energy ($E_C)_{min}$ for neutrons of zero kinetic energy is given by

$$(E_C)_{min} = M_X + M_n - M_C = S_n$$

$$= (M_{\text{XO}} + M_n - M_C) + \delta_x - \delta_C \quad \ldots(14.6-7)$$

where $S_n = M_X + M_n - M_C$ is the neutron separation energy of the compound nucleus C. In the second line on the r.h.s. of Eq. (14.6-7) we have written the masses $M_X$ and $M_C$ to show the pairing energy terms explicitly:

$$M_X = M_{\text{XO}} + \delta_x, \quad M_C = M_{\text{CO}} + \delta_C$$

Here $M_{\text{XO}}$ and $M_{\text{CO}}$ include all the terms of the semi-empirical formula, excluding the pairing energy terms.

Writing $E_{CO} = M_{\text{XO}} - M_n - M_{\text{CO}}$, we then get

$$(E_C)_{min} = E_{CO} + (\delta_x - \delta_C)$$

$(\delta_x - \delta_C)$ can be positive or negative, depending on the nature of the target nucleus (i.e., whether it is $e^-e^-$, $e^-e^+$ etc.) In Table 14.3 we list the different possibilities for neutron-induced fission:

Table 14.3

<table>
<thead>
<tr>
<th>Target nucleus (X)</th>
<th>Compound nucleus (C)</th>
<th>$\delta_x$</th>
<th>$\delta_C$</th>
<th>$\delta_x - \delta_C$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even-even</td>
<td>Even-odd</td>
<td>$&lt; 0$</td>
<td>$0$</td>
<td>$&lt; 0$</td>
<td>No thermal fission</td>
</tr>
<tr>
<td>Even-odd</td>
<td>Even-even</td>
<td>$0$</td>
<td>$&lt; 0$</td>
<td>$&gt; 0$</td>
<td>Thermal fission possible</td>
</tr>
<tr>
<td>Odd-even</td>
<td>Odd-odd</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>No thermal fission</td>
</tr>
<tr>
<td>Odd-odd</td>
<td>Odd-even</td>
<td>$&gt; 0$</td>
<td>$0$</td>
<td>$&gt; 0$</td>
<td>Thermal fission possible</td>
</tr>
</tbody>
</table>

From the table it is seen that the net effect of the pairing energy terms is to reduce the excitation energy of the compound nucleus, if the target nucleus is even-even (e.g., $^{233}\text{Th}$ or $^{238}\text{U}$). Hence additional energy must be supplied in the form of the kinetic energy of the neutrons $E_n$ so that the excitation energy $E_C$ may exceed the activation energy $E_f$: 
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\[ E_C = (E_C)_{max} + E_n > E_f \]  \( \quad \text{(14.6-8)} \)

So only fast neutrons can induce fission in these nuclei. This is also true for odd-even target nuclei (e.g., \(^{233}\text{Np}\)).

On the other hand, if the target nucleus is even-odd, then the net effect of the pairing energy terms is to increase \( E_C \) which may even exceed the activation energy for these nuclei (as in the case of \(^{233}\text{U}, ^{235}\text{U} \) and \(^{239}\text{Pu}\)). So these nuclei undergo thermal neutron fission. This is also the case with odd-odd target nuclei e.g., \(^{242}\text{Am} \) with \( Z = 95 \). These are in agreement with the observations made in the last column of Table 14.3.

### 14.7 Bohr-Wheeler theory of nuclear fission

N. Bohr and J.A. Wheeler put forward the theory of nuclear fission based on the liquid drop model of the nucleus (1939)*.

It is possible to calculate the activation energy \( E_f \) for fission of different nuclei (see Table 14.2) on the basis of this theory.

If mechanical vibrations are set up within a liquid drop, it can lead to the break-up of the drop. To do this, energy must be supplied from outside. Since an atomic nucleus behaves like a charged liquid drop (see Ch. IX), similar vibrations may be generated in it if it gains some excitation energy which is possible if, for instance, the nucleus absorbs a neutron.

The vibrations set up in the nucleus deform it due to which its surface energy \( E_s \) and electrostatic energy \( E_C \) are both changed. The sum of these two energies for a spherical nucleus is given by Eq. (14.7-1) below.

\[ E_0 = E = (E_1 + E_2) = \frac{2}{3} a_2 A^{2/3} + 4 \pi \frac{Z^2}{r_0 A^{1/3}} \]

\[ = 4 \pi \frac{Z^2}{r_0 A^{2/3}} + \frac{3}{5} \frac{Z^2}{4 \pi \varepsilon_0 r_0 A^{1/3}} \]

\[ \quad \text{...(14.7-1)} \]

where \( S \) is the surface energy per unit area. For the two fragments we have also to take into account their electrostatic energy when they are just separated. The distance between their centres is then \( 2R' \) where \( R' = r_0 (A/2)^{1/3} \) is the radius of each fragment, which are assumed to be spherical of equal mass and charge. So the energy after separation is

\[ E = 2 a_2 (A/2)^{2/3} + 2 a_1 \frac{(Z/2)^2}{(A/2)^{1/3}} + \frac{(Ze/2)^2}{4 \pi \varepsilon_0 \times 2 r_0 (A/2)^{1/3}} \]

\[ = 2 \times 4 \pi \frac{Z^2}{r_0 (A/2)^{2/3}} S \]

\[ + \frac{1}{4 \pi \varepsilon_0} \left\{ \frac{3}{5} \frac{(Ze/2)^2}{r_0 (A/2)^{1/3}} + \frac{(Ze/2)^2}{2r_0 (A/2)^{1/3}} \right\} \]

\[ \quad \text{...(14.7-2)} \]

Hence the critical energy of deformation to cause fission is (neglecting the energy of the neck)

\[ \Delta E_{ct} = E - E_0 \]

\[ = 4 \pi \frac{Z^2}{r_0 A^{2/3}} S \left( 2^{1/3} - 1 \right) \]

\[ + \frac{1}{4 \pi \varepsilon_0} \frac{3}{5} \frac{r_0 (A/2)^{1/3}}{2 - 1 + \frac{5}{3} \frac{2^{1/3}}{8}} \]

\[ \quad \text{...(14.7-3)} \]

Writing

\[ y = \frac{\Delta E_{ct}}{4 \pi \frac{Z^2}{r_0 A^{2/3}} S} \]

\[ \quad \text{...(14.7-4)} \]

and

\[ x = \frac{3\pi e^2}{2 \times 4 \pi \varepsilon_0 \times 5 r_0 A^{1/3} \times 4 \pi \frac{Z^2}{r_0 A^{2/3}} S} \]

* It may be noted that the Russian physicist Y. Frenkel had also proposed a similar theory about the same time.
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This expression contains terms in $\alpha_4$ in addition to those in $\alpha_2$, since there is strong coupling between the second and fourth modes of vibration. The minimum energy of deformation can be found by minimizing Eq. (14.7-11) w.r.t. $\alpha_4$ which gives $\alpha_4 = -243 \alpha_2^2/595$. So the minimum energy can be expressed in terms of $\alpha_2$ alone.

Since the $\alpha$'s are small numbers, the main contribution to the deformation energy comes from the terms containing $\alpha_2^2$ and we get

$$\Delta E = \alpha_2^2 \left( \frac{2}{5} \alpha_2 A^{2/3} - \frac{1}{5} \alpha_3 \frac{Z^2}{A^{1/3}} \right)$$

...(14.7-12)

For $Z$ sufficiently large, $\Delta E$ will become negative which means that spontaneous fission will occur instantaneously. The limiting condition for this to happen is

$$\frac{Z^2}{A} > \frac{2 \alpha_2}{\alpha_3}$$

...(14.7-13)

We substitute the values of $\alpha_2$ and $\alpha_3$ and write

$$(Z^2/A)_\text{lim} = \frac{2 \alpha_2}{\alpha_3} \approx 4 \pi \varepsilon_0 \times \frac{40 \pi r_0^3 S}{\varepsilon^2}$$

...(14.7-14)

We see that instantaneous spontaneous fission would occur if $(Z^2/A) > (Z^2/A)_\text{lim}$. Thus the condition $(Z^2/A) > (Z^2/A)_\text{lim}$ sets the limit for a nucleus to be stable against S.F.

Substituting the values of the parameters $\alpha_2$ and $\alpha_3$ in Eq. (14.7-14), we get

$$(Z^2/A)_\text{lim} = 50$$

So nuclei with $Z^2/A > 50$ will be unstable against S.F. This should be compared with the value $Z^2/A = 35.6$ for $^{238}$U which is stable against instantaneous spontaneous fission. The limiting value is reached for a nucleus in the transuranic region with $Z = 117$ and $A = 270$ for which $Z^2/A = 50.7$. Because of the shell structure, this might actually be shifted towards smaller $Z$.

Bohr and Wheeler have calculated the critical deformation $(\alpha_2)_\text{cr}$ for which the deformation energy $\Delta E$ is maximum. In this case

$$\Delta E_{\text{cr}} = 4 \pi \varepsilon_0 \frac{3}{4} \frac{S}{r_0 A^{1/3}} \left( \frac{98}{135} (1-x)^3 + \frac{11368}{34425} (1-x)^4 + \ldots \right)$$

...(14.7-15)

Here $x = \frac{Z^2/A}{(Z^2/A)_\text{lim}}$ which is given by Eq. (14.7-5).

The values of the activation energy $E_f$ given in Table 14.2 are actually those of $(\Delta E)_{\text{cr}}$ as given above.
In Fig 14.9 is shown the plot of $y = F(x) = \frac{\Delta E_{cr}}{4 \pi r^2 \times A^{2/3}}$ calculated on the basis of Eq. (14.7-6) for light nuclei and Eq. (14.7-15) for heavy nuclei. The intermediate region is drawn by interpolation.

![Figure 14.9](Image)

**Quantum effects**

The classical theory discussed above neglects two quantum effects: (1) Effect due to barrier penetration problem, when fission takes place at excitation energies below the fission threshold; (2) Existence of zero-point energy in the vibration of the deformed drop.

If we assume an inverted one-dimensional harmonic oscillator type of barrier $\left( -\frac{1}{2} \frac{d^2 E}{dx^2} \right)$ where $B$ is the inertia parameter, $\alpha_i$ is the deformation parameter and $\omega_0$ is the circular frequency of vibration of the deformed drop, then the Gamow barrier penetration factor can be written as

$$P = \left( 1 + \exp \left\{ \frac{-2\pi(E - E_f)}{\hbar \omega_0} \right\} \right)^{-1}$$

...(14.7-16)

At some value of $\omega_0$, the nucleus is in the saddle point configuration for which $E = E_f$. This gives $P = 1/2$ and

$$\hbar \omega_0 = \hbar \left[ \frac{5 E_{20}}{4 \pi B_2} \frac{\partial^2 E_f}{\partial \alpha_2^2} \right]^{1/2}$$

...(14.7-17)

considering only second order vibrations for which $B = B_2$ and $\alpha = \alpha_2$. For $A \sim 240$, $\hbar \omega_0 \sim 1$ MeV. The spontaneous fission life time is then given by

$$\frac{1}{\tau} = 2\pi \omega_0 \exp \left\{ \frac{-2\pi(E_f - E)}{\hbar \omega_0} \right\}$$

...(14.7-18)

G.T. Seaborg calculated $E_f$ from $t$ vs. $Z^2/A$ plot for different heavy elements and found the fission life-time to be given by (in seconds)

$$t = 10^{-21} \times 10^{17.8 - 3.75(Z^2/A)}$$

...(14.7-19)

This is in contrast to the expression obtained by S. Frenkel and N. Metropolis on the basis of barrier penetration probability which gives (in seconds)
t = 10^{-21} \times 10^{7.85} \epsilon_f 

...(14.7-20)

When Seaborg used these results, he found 

\[ E_f = 19.0 - 0.36 Z^2 / A \]

...(14.7-21)

This formula gives fairly good agreement with the experimental results for a narrow range of nuclei.

The zero-point energy calculated from the shape of the crater near zero deformation has a value \( \sim 0.4 \text{ MeV} \) small compared to the excitation involved which justifies classical considerations.

**Comparison with experiment:**

Agreement between the theoretically calculated value of \( E_f (\sim 15 \text{ MeV}) \) on the basis of Bohr-Wheeler theory and experimental values (\( \sim 6 \text{ MeV} \)) is very poor. It shows that the activation energy \( E_f = (\Delta E)_a \) is not as sensitive to changes in \( x \) in Eq. (14.7-15). Quantum corrections discussed above do not make much difference. Observed threshold energies are not smooth functions of \( x \). Further, the liquid drop model does not explain asymmetric fission, which is one of its major drawbacks (see §14.10).

### 14.8 Cross section of neutron induced fission

In Fig. 14.11 is shown the variation of the cross section \( \sigma_f \) of neutron induced fission with the neutron energy \( E_n \). At very low energies (e.g., for thermal neutrons), the cross section is very high (\( \sim 600 \text{ barns} \)). As \( E_n \) increases, \( \sigma_f \) decreases according to the \((1/v)\) law. Between about 1 eV to 100 eV, resonance peaks appear in the cross section curve. At higher energies, \( \sigma_f \) again decreases monotonically attaining values of the order of 1 barn at \( E_n \sim 1 \text{ MeV} \).

![Graph of fission cross section vs energy](image)

**Fig. 14.11. Variation of fission cross section with neutron energy.**

As seen in § 14.5, even-odd nuclei e.g., \(^{235}\text{U}, ^{239}\text{Pu}\) etc. undergo thermal neutron fission for which the cross section \( \sigma_f \) is very high.

### 14.9 Fertile materials

Of the three isotopes which are fissionable with thermal neutrons, only \(^{235}\text{U}\) is naturally occurring with an abundance of 0.71% in natural uranium. The other two thermally fissile isotopes \(^{233}\text{U}\) and \(^{239}\text{Pu}\) do not exist in nature, but can be produced artificially. They have the half-lives \(1.6 \times 10^9\) y and \(2.41 \times 10^4\) y respectively. Because of their relatively long half-lives, they can be produced in substantial amounts in nuclear reactors. Notice that all these three isotopes are of the even-odd type and have high cross sections for thermal neutrons fission (see Table 14.4). All of them can be used as fuel in thermal neutron reactors.

\(^{239}\text{Pu}\) and \(^{233}\text{U}\) can be produced in nuclear reactions, starting from \(^{238}\text{U}\) and \(^{232}\text{Th}\) respectively. These two even-even isotopes are not fissionable with thermal neutrons. They are known as fertile materials since they can be converted into the above thermally fissile isotopes. The relative abundances of the above two fertile isotopes are quite high, being 99.3% for \(^{238}\text{U}\) in natural uranium and 100% for \(^{232}\text{Th}\).

\(^{238}\text{U}\) by absorbing slow neutrons is converted into the isotope \(^{239}\text{U}\) by \((n, \gamma)\) reaction. The latter, by two successive \(\beta^-\) decays, is transformed into the isotope \(^{239}\text{Pu}\) (\(Z = 94\)) which is a transuranic element. The equations for these transformations are:

\[
\begin{align*}
^{238}_{92}\text{U} + \beta^- & \rightarrow ^{239}_{92}\text{U} + \gamma \\
& (\tau = 23.5 \text{ min})
\end{align*}
\]
14.10 Particle induced fission and photo-fission

Fission can be induced in heavy elements by agents other than neutrons. Thus fission has been observed in some elements by bombardment with protons, deuterons and α-particles. Of course in these reactions, there is the problem of Coulomb barrier penetration by the incident particle. \((p, f)\) reaction has been observed in uranium by 7 MeV protons, while somewhat higher energy deuterons are required to induce \((d, f)\) reaction in uranium. The latter reaction is actually a direct reaction in which the neutron from the incident deuteron enters the nucleus to cause fission, while the proton moves along almost in the forward direction (see Ch. XI). So it is in reality a neutron induced fission. The compound nucleus, in this case, has a lower excitation energy than in a regular thermal neutron induced reaction. So the fission threshold can be measured conveniently by utilizing this reaction.

Photo-fission induced by high energy γ-rays photons has been observed in some heavy elements. For \(^{238}\text{U}\), the \((γ, f)\) reaction shows a sharp threshold at 5.1 MeV. The cross sections are small (\(\sim 10^{-7}\text{b}\)).

Asymmetry in fission:

Calculation of the fission barrier height \(E_b\) as a function of the deformation parameter shows that the minimum value of \(E_b\) corresponds to symmetric fission and hence symmetric fission should be more probable than asymmetric fission, from the point of view of the liquid drop model. The reason why this is not so can be understood as follows. The initial deformation prior to fission is symmetric, as required by the liquid drop model. After the saddle point is attained, the disintegration into the fission fragments do not follow immediately, but takes place only after the nucleon shells in the would-be fragments have been formed. The numbers of nucleons in these shells happen to be different and hence asymmetric fission occurs. The time scales involved in the two processes can be estimated as follows.

The fission time is \(\tau_f = \delta/v\) where \(\delta\) is the distance between the flying fragments (\(\sim 10^{11}\text{m}\)) and \(v\) their velocity (\(\sim 10^7\text{m/s}\)) so that \(\tau_f \sim 10^{-20}\text{s}\).

The shell formation time, on the other hand is \(\tau_s \sim \hbar/\Delta E_s\), where \(\Delta E_s \sim 1\text{MeV}\) is the mean separation between the one-particle levels. This gives \(\tau_s \sim 10^{-34}/10^{-13} = 10^{-21}\text{s} < \tau_f\).

14.11 Shell effect on nuclear fission; Shape isomerism

If the spontaneous fission \((S.F.)\) half-lives for different isotopes are plotted as functions of \(Z^2/A\), elementwise, we get curves of the type shown in Fig. 14.12. Though there are individual variations, there is a general trend for \(S.F.\) half-lives to decrease rapidly as \(Z^2/A\) increases. Correspondingly the probability of \(S.F.\) increases rapidly with increasing \(Z^2/A\). The points shown in the figure are experimental data, while the solid line across the figure has been calculated according to the liquid drop model. However, some discrepancies have been noticed, which calls for the correction of the results of the liquid drop model on the basis of the nuclear shell model.

Fig. 14.12. Spontaneous fission half-lives of different isotopes as function of \(Z^2/A\).

It was first observed by the Russian scientist S.M. Polikanov and his associates (1961) that there were some spontaneously fissioning nuclei in excited states (isomers) with half-lives too short to be compatible with barrier penetration starting from an excited nucleus with normal deformations. As an example the isomeric states of some americium
isotopes \((Z = 95)\) have S.F. half-lives of a few milliseconds, which are far less than the half-lives expected on the basis of the liquid drop model. The isomeric states have excitation energies of about 2 to 3 MeV and low spin. Normally they should decay by radiative transitions (isomer transition) to ground states like so many known light nuclei isomeric states. Instead they undergo S.F. much more readily. This puzzling result can be understood by introducing shell correction to the potential energy curves. It was first pointed out by Flyorov (1966) and later substantiated by V.M. Strutinsky (1967) and also by Nilsson that there were regular modulations of the potential energy curves due to the shell effect.

It is well-known that there are gaps in the single-particle spectra for spherical nuclei for certain \(N\) and \(Z\) values. Other gaps and bunchings develop at large deformations, particularly for shapes with high symmetry, e.g., those with major to minor axis ratio of \(2:1\). These shell effects give rise to a second minimum in the potential energy surface associated with spontaneous fission isomerism. This is shown in Fig. 14.13. Such a potential has been called a double humped potential.

![Double humped potential](image)

The double humped potential shows two minima. The first represents a stable and smaller deformation predicted by the collective model, while the second represents a greater deformation. Taking the case of the 2.56 MeV isomer of \(^{238}\text{U}\), the inner and outer barrier heights are found to be both \(\approx 6\text{ MeV}\) (from direct reaction studies). Thus the modulation at the second minimum due to the shell effect is over 3 MeV.

The isomeric state excited in the second valley shown in Fig. 14.13 can either decay by spontaneous fission or by radiative transition \((\gamma)\) to the ground state in the first valley. For both these, barrier penetration is involved. However, for the fission, the barrier to be penetrated is quite thin compared to the case when barrier penetration through a single humped potential, without shell correction, is to take place. Further, the height of the second barrier is much lower. Hence the probability of fission is much larger. Correspondingly, the half-life is much shorter, in agreement with observation. The radiative transition has a low probability, not because of large difference in the angular momenta between the ground and excited states, but because of the existence of the barrier between the two wells. Hence, the fission process is competitive with the radiative transition from the isomeric state.

**14.12 Nuclear fusion and thermonuclear reaction**

It may be noted that it is the isomeric state which sees the second hump while undergoing fission and consequently has to penetrate a much thinner barrier. The fission from the ground state can take place by penetrating through the much thicker barrier (see figure) and hence is much inhibited. As an example, for \(^{242}\text{Am}\), the fission half-life for the isomeric state is only 0.014 s as compared to \(\approx 10^3\) s for the fission of the ground state, being lower by a factor of \(\approx 10^3\). A few examples of similar isomeric fission are given below:

- \(^{236}\text{U}^*, \ 238\text{U}^*, \ 237\text{Np}^*, \ 235-243\text{Pu}\)
- \(^{237\text{-}240}\text{Am}\) and \(^{240-245}\text{Cm}\)

More than 40 fission isomers are now known to exist with S.F. half-lives ranging from 5 ps to 14 ms. Ratio as high as \(10^6\) has been observed in some cases between the S.F. half-life of the ground state to that of the isomeric state.

The isomeric states for which fission half-lives are much smaller than those for ground state fission are known as shape isomers and the phenomenon as shape isomerism. In normal isomerism, increased life-time of radiative decay is due to small energy or large spin change or both (see Ch. VI). For the shape isomers, it is the existence of double humped potential which is responsible for the increased probability of fission transition, comparable to that of the radiative transition. The energies of the isomeric states are also much higher (\(\approx 2\) to 3 MeV) than in the case of normal isomerism.

The existence of a second minimum in the potential energy curve gives rise to different types of resonance structures in the energy dependence of the fission probability of the excited nuclei. The couplings of the vibrational levels of the second well (spacing \(\approx 1\) MeV) to the states of the first well, give rise to gross structure resonances. The states of second well at the first well excitation energies of the order of the neutron binding energies and having separation of \(\approx 0.1\text{ keV}\) can couple with the densely packed first well levels. This can increase the fission probability of the corresponding first well levels, which are degenerate with the second well levels, giving rise to intermediate structure resonance (‘door way’ states).

After a large number of experiments on the determination of the parameters of the double humped potential, the following qualitative picture has emerged:

- \((E_0)_1 > (E_0)_2\) for Pu, Am and Cf;
- \((E_0)_1 < (E_0)_2\) for Th and U;
- \(E_c = 2\) to 3 MeV.

**14.12 Nuclear fusion and thermonuclear reaction**

The practical utilization of the energy released in nuclear fission, both for beneficial and destructive purposes, is now an accomplished fact (see Ch. XV). Concerted efforts are now being made all over the world for the practical use of the energy released in another type of nuclear
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reaction for peaceful purposes. This reaction is known as the fusion reaction. As the name implies, in this type of reaction, two (or more) light nuclei fuse together to produce a heavier nucleus. For very light nuclei, such reactions are usually exoergic, which can be understood qualitatively from the binding fraction curve given in Ch. II (see Fig. 2.2). For very light nuclei, the binding fraction \( f_a \) is a rapidly rising function of \( A \) which means that a nucleus produced as a result of the fusion of two lighter nuclei may have greater binding energy than the combined binding energies of the latter.

For example, if we consider the fusion of two deuterons to produce an \( \alpha \)-particle according to the equation \( ^2\text{H} + ^2\text{H} \rightarrow ^4\text{He} + \alpha \), we get the \( Q \) value of the reaction as

\[
Q = B_\alpha - 2B_d = 4f_{\text{He}} - 2 \times (2f_{\text{He}}) = 28.3 - 2 \times 2.225 = 23.84 \text{ MeV}
\]

Thus the energy released per nucleon in this reaction is 23.84/4 = 5.96 MeV. This is much smaller than the energy released per nucleon in fission. The latter is about (200/238) or 0.84 MeV. So mass for mass, the fusion reaction usually gives more energy than the fission reaction.

It may be noted that the fusion reaction given in the example above does not actually occur. Due to the huge quantity of energy release, the \(^4\text{He} \) nucleus has so great an excitation energy that it breaks up by the emission of a proton or a neutron, as soon as it is formed, giving rise to the following reactions:

\[
\begin{align*}
^2\text{H} + ^2\text{H} &\rightarrow ^3\text{He} + \alpha \\
^3\text{He} + ^2\text{H} &\rightarrow ^4\text{He} + \alpha
\end{align*}
\]

Other examples of exoergic fusion reactions are

\[
\begin{align*}
^3\text{He} + ^2\text{H} &\rightarrow ^4\text{He} + \alpha \\
^4\text{He} + ^2\text{H} &\rightarrow ^6\text{He} + \alpha \\
^6\text{Li} + ^2\text{H} &\rightarrow ^8\text{He} + \alpha \\
^7\text{Li} + ^2\text{H} &\rightarrow ^9\text{He} + \alpha
\end{align*}
\]

The energy released in the last four reactions is much greater than in the first two. The energy released in the \( d-t \) reaction is 17.6 MeV or 3.5 MeV per nucleon.

The fusion reactions in the examples given above can occur only if the incident projectile (usually the deuteron) has sufficient high energy so that it can come into close contact with the target nucleus by overcoming the electrostatic repulsion. For two deuterons the electrostatic energy, when in contact \( (R_s = 4.3 \times 10^{-13} \text{ m}) \) is

\[
V_s = \frac{e^2}{8 \pi \varepsilon_0 R_s} = 0.17 \text{ MeV}
\]

Classically the fusion can take place only if the incident particle has energy at least equal to \( V_s \). However, quantum mechanically there may be tunnelling through the barrier, so that the reaction may take place at much lower energy (~ a few keV). Of course the probability of penetration through the barrier will be higher at higher energies.

Reactions using charged particles as projectiles are carried out in the laboratory, using particle accelerators. However, there is another way in which the above fusion reactions may be made to occur. The incident particles \( (p, d \text{ etc.}) \) can acquire fairly high energies if the gases e.g., hydrogen or deuterium are heated to very high temperatures (several hundred million degrees). According to the kinetic theory of gases, the mean thermal energy of the gas molecules increases as the temperature increases. At the temperature \( T \) K, the mean thermal energy is of the order

\[
E = kT = 0.87 \times 10^{-4} \text{ TeV}
\]

(14.12-7)

Here \( k = 1.38 \times 10^{-23} \text{ J/degree} \) is the Boltzmann constant. For \( T = 10^7 \text{ K}, E \) is of the order of a keV.

If a gas is heated to such a high temperature, many changes will take place in the structure of the molecules and atoms of the gas. At the temperature of a few thousand degrees kelvin, the molecules of the gas dissociate and the bare atoms move about freely at random. If the temperature is raised to \( 10^6 \text{ K} \) or higher, then the electrons in the orbits of the atoms begin to separate from the latter, thereby ionising them. The process, known as thermal ionization is governed by Saha ionization equation deduced by the Indian physicist M.N. Saha. Eq. (14.12-7) shows that the energies of the atoms are then of the order of a few electrons volts, i.e., comparable to the ionization energy of the atoms. Thus, at these temperatures, the positively charged atomic ions and the negative electrons move about freely. They constitute what is known as a plasma, which is regarded as the fourth state of matter. The plasma as a whole is electrically neutral. So unless electric or magnetic fields are applied, the only force acting on the plasma is the gravitational force.

As the temperature of the plasma increases all the electrons in the atoms get loose from the latter and we get a mixture of bare nuclei and electrons. As the temperature further rises, the mean thermal energy of the nuclei and the electrons increases. When the temperature is \( 10^7 \text{ K} \) or higher, this energy is of the order of kilo electron volts or higher. The energy distribution of the plasma particles is similar to the Maxwellian distribution. Hence the plasma contains some particles which have much higher energies than the mean thermal energy \( kT \) for a particular temperature \( T \). At higher temperatures, the number of the high energy particles increases. Such higher energy particles (nuclei) have finite probability of overcoming the mutual Coulomb repulsion between them which makes possible for them to penetrate through the potential barrier and produce fusion reaction. This is known as the thermo-nuclear reaction.

14.13 Source of energy in stars

Thermo-nuclear reactions discussed in the previous section is of special interest, because of their possible use in large scale energy release
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on the earth, as also because of their involvement in astrophysical phenomena.

It is known that the sun is an average sized star, emitting $4 \times 10^{38}$ J energy per second. Its mass is $2 \times 10^{30}$ kg, which means that the mean energy production in the sun is $2 \times 10^{38}$ J/kg.s. Actually the rate of energy liberation in the central region, where the energy is produced, must be larger than the above figure. The energy liberation in the sun is going on at the above rate for at least $4 \times 10^9$ years.

Neither chemical reactions nor gravitational energy changes can account for the above huge rate of energy release for such a long time. For example, if the sun consisted of carbon and oxygen only and the solar energy were generated due to the burning of carbon, then all of it would burn up in only 1500 years. That leaves nuclear energy as the only possible source of solar (and stellar) energy.

The sun is known to be mainly made up of hydrogen and helium (90%) in about equal proportions. If by some suitable series of nuclear reactions, four hydrogen nuclei combine to produce one helium nucleus, then the energy release for such each fusion will be

$$E = 4M_H - M_{He} = 4 \times 1.007825 - 4.002603 = 0.028697 \text{u} = 26.73 \text{ MeV} = 4.28 \times 10^{-12} \text{ J}$$

Since each kilogram of hydrogen contains about $6 \times 10^{26}$ protons, the energy content of such a source will be about $2.4 \times 10^{35}$ J/kg, which could liberate energy at the rate of $2 \times 10^{34}$ J/kg.s for $10^9$ years. So the nuclear reactions leading to the fusion of four hydrogen nuclei to produce one helium nucleus could be adequate for the energy production in the sun.

R. Atkinson and F. Houtermans (1928) were the first to suggest that the successive capture of four protons by some light nuclei to produce an $\alpha$-particle might be the processes which would release energy at reasonable rates for the sun to continue burning for such a long time. They had suggested the idea of cyclic nuclear reactions, the importance of which was proved later.

Two such thermonuclear reaction cycles have been suggested for the production of a helium nucleus by the fusion of four protons.

(A) Proton-proton cycle:

1. $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + ^1\pi + \pi + 0.42 \text{ MeV}$
2. $^1\text{H} + ^2\text{H} \rightarrow ^3\text{He} + ^1\gamma + 5.5 \text{ MeV}$
3. $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{He} + 12.8 \text{ MeV}$

Two reactions, each of (1) and (2), must occur for each reaction (3) to take place. When these are written out in detail and all the reactions are added, we get

$$2^1\text{H} + 2^1\text{H} + 2^3\text{He} + 3\text{He} \rightarrow 2^2\text{H} + 2^\pi + 2^1\text{H} + 2^1\pi + 2^2\text{H} + 24.64 \text{ MeV}$$

So the net result is

$$^4\text{He} \rightarrow ^4\text{He} + 2^\pi + 2^\pi + 2^\gamma + 24.64 \text{ MeV} \quad (14.13-1)$$

(B) Carbon cycle:

This cycle was proposed by H.A. Bethe (1939) and comprises of the following reactions:

1. $^{12}\text{C} + ^1\text{H} \rightarrow ^{13}\text{N} + ^1\gamma + 1.95 \text{ MeV}$
2. $^{13}\text{N} \rightarrow ^{13}\text{C} + ^1\pi + ^1\gamma + 2.22 \text{ MeV}$ (=$\tau = 10 \text{ min}$)
3. $^{13}\text{C} + ^1\text{H} \rightarrow ^{14}\text{N} + ^1\gamma + 7.54 \text{ MeV}$
4. $^{14}\text{N} + ^1\text{H} \rightarrow ^{15}\text{O} + ^1\gamma + 7.35 \text{ MeV}$
5. $^{15}\text{O} \rightarrow ^{15}\text{N} + ^1\pi + ^1\gamma + 2.7 \text{ MeV}$ (=$\tau = 2 \text{ min}$)
6. $^{15}\text{N} + ^1\text{H} \rightarrow ^{12}\text{C} + ^4\text{He} + 4.96 \text{ MeV}$

The net result is

$$^{12}\text{C} + 4^1\text{H} \rightarrow ^{12}\text{C} + ^4\text{He} + 2^1\pi + 2^\pi + 2^\gamma + 26.72 \text{ MeV} \quad (14.13-2)$$

Notice that the net result is the fusion of four protons to produce one $^4\text{He}$ nucleus in the presence of $^{12}\text{C}$, which must be present, but is not destroyed in the cycle. So it acts like a catalyst. Since about $1.7 \text{ MeV}$ energy is carried away by the neutrinos which escape, the net-energy release is $25.02 \text{ MeV}$. The reaction period of the cycle is determined essentially by the reaction period of the reactions (1) and (4) which are 2.5 x 10$^9$ y and 3 x 10$^9$ years respectively.

The present evidences are that for the stars with masses between 0.4 to 2.5 solar masses, the main part of the energy production is due to the

* See Theory of Atomic Nucleus and Nuclear Energy Sources by G. Gamow and C.L. Crichel.
carbon cycle rather than the p-p cycle. For less massive stars, the situation is reversed. Since according to astronomical data, stars of 0.4 or lower solar masses form the bulk of the stellar population in our galaxy, we can say that whereas carbon cycle probably goes on in some very bright stars, the majority of the stars derive their energy from the p-p cycle.

Thermo-nuclear fusion opens up immense possibilities for the peaceful utilization of nuclear-energy, in which the sea water may serve as the main and an almost inexhaustible source of fuel. The technological problems involved are highly complicated and are as yet far from being solved. Some inklings of the attempts being made in various countries to achieve a break-through will be briefly discussed in Ch. XV.

Though the peaceful utilization of the fusion energy still remains to be achieved, harnessing of this energy for destructive purposes in the front of the hydrogen bomb was made possible nearly forty years ago.

14.14 Nucleo-synthesis

The thermo-nuclear fusion reaction has an important bearing on another astrophysical problem, viz., the origin of the chemical elements or nucleosynthesis.

The nature of the distribution of the chemical elements in the sun and in other stars belonging to the main sequence shows that the lighter elements like hydrogen, helium etc. have much greater abundances than those of the heavier elements. As shown in Fig. 14.14, the relative abundances of the elements show, on the average, rapid decrease with increasing mass number (A) for the lighter elements and become almost constant for \( A > 100 \). However, there are local variations. Thus the abundance of deuterium is unusually low, which is also true for a few other lighter elements like lithium, beryllium and boron. On the other hand, in the case of iron \( (Z = 26) \) the relative abundance is unusually high. In Fig. 14.14, this appears as a sharp peak, known as the iron peak at \( A = 56 \).

There are also few double peaks at the mass numbers \( A = 80 \) and \( 90; 130 \) and \( 138; 196 \) and \( 208 \), close to the neutron magic numbers \( 50, 82 \) and \( 126 \). Any theory of nucleo-synthesis must account for these observed features of the abundance curve.

An important point to note is that these abundances are in no way correlated with the chemical properties of the elements. On the other hand, there is a clear correlation between the cosmic abundances of the elements and their nuclear properties.

There seems to be fairly close resemblance between the elemental compositions of such diverse objects in the universe as the sun, the stars in our own galaxy (Milky way) and probably the stars in other galaxies (see Table 14.5). The case of the earth and other minor planets nearer the sun is somewhat different. Because of the relatively weak gravitational fields and higher temperatures, very light elements like hydrogen and helium have escaped from their atmosphere. In the case of the larger planets (e.g., Jupiter, Saturn etc.) far away from the sun, the gravitational fields are much stronger and the temperatures lower. So the above lighter elements are still to be found in them in relatively greater abundance.

A better index is the isotopic abundance ratios of the different elements. These are found to be almost the same in the earth's crust, in certain meteorites (carbonaceous condrite), in the sun and in the stars. This implies common nuclear reaction origin of the elements. The chemical effects cannot vary the isotopic ratios appreciably.

Again the earth's crust is composed of elements in which the even-even stable nuclei predominate (> 86%). Of these, the nuclei with doubly closed shells (doubly magic) are in preponderance (more than half). Examples are oxygen \( (99.7 \% \ 16\text{O} \text{w} Z = 8, N = 8) \), silicon \( (92.3\% \ 28\text{Si} \text{w} Z = 14, N = 14) \), calcium \( (97\% \ 40\text{Ca} \text{w} Z = 20, N = 20) \). Correlation with nuclear property is thus in clear evidence, since (i) the even-even stable nuclei are more than 60% in number amongst all stable nuclei; (ii) doubly magic nuclei being the most stable, these isotopes have relatively much greater abundances.
Table 14.5

<table>
<thead>
<tr>
<th>Relative abundances of chemical elements</th>
<th>Stars</th>
<th>Sun</th>
<th>Earth's crust</th>
<th>Stony meteorites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>11.4</td>
<td>11.5</td>
<td>8.3</td>
<td>6.9</td>
</tr>
<tr>
<td>Helium</td>
<td>10.2</td>
<td>10.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Carbon</td>
<td>6.4</td>
<td>7.4</td>
<td>6.3</td>
<td>6.1</td>
</tr>
<tr>
<td>Oxygen</td>
<td>8.0</td>
<td>9.0</td>
<td>8.5</td>
<td>8.4</td>
</tr>
<tr>
<td>Sodium</td>
<td>7.1</td>
<td>7.2</td>
<td>7.3</td>
<td>6.4</td>
</tr>
<tr>
<td>Magnesium</td>
<td>7.5</td>
<td>7.8</td>
<td>7.2</td>
<td>7.7</td>
</tr>
<tr>
<td>Aluminium</td>
<td>6.9</td>
<td>6.4</td>
<td>7.8</td>
<td>6.8</td>
</tr>
<tr>
<td>Silicon</td>
<td>7.5</td>
<td>7.3</td>
<td>8.2</td>
<td>7.8</td>
</tr>
<tr>
<td>Potassium</td>
<td>5.3</td>
<td>6.8</td>
<td>7.1</td>
<td>5.6</td>
</tr>
<tr>
<td>Calcium</td>
<td>6.7</td>
<td>6.7</td>
<td>7.2</td>
<td>6.5</td>
</tr>
<tr>
<td>Titanium</td>
<td>6.0</td>
<td>5.2</td>
<td>6.4</td>
<td>5.3</td>
</tr>
<tr>
<td>Chromium</td>
<td>4.9</td>
<td>5.0</td>
<td>5.2</td>
<td>?</td>
</tr>
<tr>
<td>Vanadium</td>
<td>5.8</td>
<td>5.7</td>
<td>5.4</td>
<td>5.8</td>
</tr>
<tr>
<td>Manganese</td>
<td>6.5</td>
<td>5.9</td>
<td>5.6</td>
<td>5.6</td>
</tr>
<tr>
<td>Iron</td>
<td>6.7</td>
<td>7.2</td>
<td>7.2</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Let us now see how the different elements could have been cooked (synthesized) within the stars in the course of stellar evolution. We know that the internal temperature of the stars is around $10^4$ K. The internal temperature of the sun, for example, is $2 \times 10^4$ K. Such high temperatures are initially generated by the gravitational contraction. When the temperature reaches the above values, the different lighter nuclei present within the high density plasma inside the stars have sufficient kinetic energy, especially in the Maxwellian tail, to overcome their mutual electrostatic repulsion and come into close contact. They then produce thermonuclear fusion and generate enormous quantity of energy. It is this energy which keeps the stars burning for such long periods of time since their birth ($\sim 5 \times 10^9$ years in the case of the sun).

As we saw in the previous section, for the majority of the stars in the main sequence, it is the fusion of four hydrogen nuclei to produce a $^4$He nucleus which is responsible for energy generation. Two cyclic processes have been visualized for the above fusion to occur, viz., the proton-proton cycle and the carbon cycle which were discussed in the last section.

When hydrogen is used up to a large extent in the interior region, the fusion will stop. The temperature is now $\sim 4 \times 6 \times 10^7$ K. Fusion will still go on in the stellar shell. The core now contracts while the shell expands producing a red giant which happens when the stellar mass $M < 3M_\odot$, $M_\odot$ being the solar mass.

**Nuclear Fission and**

For stars with $M > M_\odot$, the depletion of the greater portion of hydrogen stops the fusion and gravitational contraction again starts, so that the temperature begins to rise. When the temperature rises to $2 \times 10^8$ K or higher, the helium nuclei begin to fuse (helium burning) producing the nuclei $^{12}$C, $^{16}$O, $^{20}$Ne and other $\alpha$-particle nuclei.

$$^4\text{He} + ^4\text{He} \rightarrow ^8\text{Be} \quad (Q = -94.2 \text{ keV})$$

$$^8\text{Be} + ^4\text{He} \rightarrow ^{12}\text{C} + \gamma \quad (Q = 7.73 \text{ MeV})$$

$$^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O} + \gamma \quad (Q = 7.16 \text{ MeV})$$

$$^{16}\text{O} + ^4\text{He} \rightarrow ^{20}\text{Ne} + \gamma \quad (Q = 4.73 \text{ MeV})$$

The first of these reactions is slightly endoergic which can however still take place at the temperature of $2 \times 10^8$ K at which the mean thermal energy is $2.6$ keV (see Eq. 14.12-7). The other reactions being exoergic occur easily.

When most of helium has been used up, helium burning stops and gravitational contraction starts once again, which raises the temperature further. At $10^9$ K, photo-nuclear reactions e.g., $^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}$ produce $\alpha$-particles with $Q = -4.73 \text{ MeV}$ in the high flux of the $\gamma$-rays of energies higher than the above $Q$. These $\alpha$-particles can then produce $(\alpha, \gamma)$ reactions to synthesize still heavier nuclei (nucleosynthesis) e.g.,

$$^{20}\text{Ne} + ^4\text{He} \rightarrow ^{24}\text{Mg} + \gamma \quad (Q = 9.3 \text{ MeV})$$

When the temperature rises further to about $3 \times 10^9$ K, burning of $^{28}\text{Si}$ starts. Due to the fusion of two $^{28}\text{Si}$ nuclei, $^{56}\text{Ni}$ ($Z = 28$) is produced ($Q = 10.9 \text{ MeV}$). Starting from this, by $\beta$- and $\beta^\prime$ emissions, $^{56}\text{Fe}$ ($Z = 26$) is produced. These processes take place in stars much heavier than sun.

In all the above cases, nuclei with mass number $A$ which are multiples of 4 are produced.

Let us now see how nuclei with mass numbers which are not multiples of 4 are produced. At $T = 3.5 \times 10^9$ K and higher, a great variety of reactions induced by $\alpha$-particles may take place in which the product nuclei with $A$ which are not multiples of 4 may be produced. These new nuclei may produce still heavier nuclei by similar reactions. Finally when an equilibrium condition for the abundance is reached for a given temperature, it is found that the elements of the iron peak are preferentially formed ($E$ process).

In some of the reactions mentioned above e.g. $(\alpha, n)$, neutrons are produced. Then neutron capture becomes a dominant process. The nuclei heavier than iron are believed to be produced in this way. Some of the products of neutron capture are radioactive. By successive $\beta$-dissintegrations they produce other nuclei.

There are two distinct situations for the neutron capture reactions to occur. In the slow or $s$ - process, the capture is slow compared to the $\beta$-decay which follows. This happens if the neutron density is moderate.
For the nuclei with a neutron shell closed, neutron capture cross section is very low. So there will be an excess abundance of the magic nuclei produced by β-decay. This accounts for the peaks at $A = 90$, 138 and 208 corresponding to the magic neutron numbers 50,82 and 126 respectively. The S-process is dominant for $A = 23$ to 46 and for $A = 63$ to 209 above the iron peak.

In the rapid or R-process, the neutron capture is much faster than the β-decay. This happens if the neutron density is very high, as in a stellar explosion. Then a β-active magic nucleus will undergo β-transformation, rather than change by neutron capture, since the latter process is highly inhibited. Thus the decay product of the magic nucleus will have higher abundance. This is the reason behind the occurrence of the second set of peaks at $A = 80$, 130 and 194 in the regions of the neutron magic numbers $N = 50$, 82 and 126 respectively.

As examples of the $R$ and $S$-processes, we shall consider the transformation of $^{56}{\text{Fe}}$ nucleus by successive neutron captures:

$^{56}\text{Fe} + ^{1}\text{n} \rightarrow ^{57}\text{Fe}$
$^{57}\text{Fe} + ^{1}\text{n} \rightarrow ^{58}\text{Fe}$
$^{58}\text{Fe} + ^{1}\text{n} \rightarrow ^{59}\text{Fe}$

$^{57}\text{Fe}$ and $^{58}\text{Fe}$ are stable, but $^{59}\text{Fe}$ is β-active with a half-life of 47 days:

$^{57}\text{Fe} \rightarrow ^{25}\text{Co}$

$^{59}\text{Co}$ is stable. If the R-process takes place, before the β decay of $^{59}\text{Fe}$, it will transform by further successive neutron capture processes:

$^{59}\text{Fe} + ^{1}\text{n} \rightarrow ^{60}\text{Fe}$
$^{60}\text{Fe} + ^{1}\text{n} \rightarrow ^{61}\text{Fe}$

The products are all β-active.

If however S-process takes place, then $^{59}\text{Fe}$ will undergo β-transformation into stable $^{59}\text{Co}$. It will not undergo neutron capture within its half-life of 47 days. So we get:

$^{59}\text{Fe} \rightarrow ^{27}\text{Co} (\text{stable})$; $^{59}\text{Co} + ^{1}\text{n} \rightarrow ^{60}\text{Fe}$

The observed abundance curve for the elements will thus differ, depending on whether the S or the R-process has been responsible for the nucleo-synthesis. The present evidence is that in the solar system, it is the S-process which is responsible.

There is a third process, known as the P-process, which gives rise to proton-rich nuclei by proton absorption.

Production of very heavy nuclei ($Z > 82$) probably occurs in the supernova explosion due to R-process in the very high neutron flux produced in the explosion. Similar effect has been observed in high altitude hydrogen bomb test which confirms this surmise. Another confirmation comes from the exponential decay of the brightness of the remnant of the supernova explosion with a half-life of 55 days. This agrees very well with the half-life for spontaneous fission of the only short half-life heavy nucleus $^{24}\text{Cf}$ ($Z=98$) near the stability line.

The observed low abundance of the nuclei of deuterium, lithium, beryllium and boron requires assumption of some special processes for their depletion. It is believed that in the hot interior of the stars, they are easily absorbed by other nuclei. Their production takes place in the cooler exterior regions of the stars (where the density is low) due to the interaction between cosmic rays and the heavy nuclei.

We now consider an important problem, the so called helium problem. Helium has a fairly large abundance (~ 25%) throughout the universe. In the course of the stellar evolution, most of helium is expected to be used up in the helium fusion processes discussed above. Why then is so much helium left in the universe? The answer to this question is believed to be linked with the origin of the universe itself.

The most promising theory about the origin of the universe is the Big Bang theory (see Ch. XVIII), originally propounded by R.A. Alpher, H.A. Bethe and G. Gamow (1948). According to this, a primordial fire-ball confined within a localized region of space, suddenly exploded about 10 to 20×10$^9$ years ago. Taking the time of the explosion to be zero time, the synthesis of the elementary particles as we know today (e.g., protons, neutrons, leptons) started about $10^4$ to $10^6$ second after the explosion. The fire ball also started expanding within a short period. The initial rate of expansion can be inferred from observations on galactic red-shift. The neutrons start decaying into protons with a half-life of about 10 min. The simple nuclei e.g., $^2\text{H}$, $^3\text{H}$, $^4\text{He}$ began to be formed at about zero plus 3 min after the explosion, due to proton-neutron capture process, followed by β-decay. These initial processes last for about a couple of hours or so, since most of the neutrons are finished by radioactive β-decay by that time as also due to the reduction in the density and temperature. Thus what we see today of the universe are the remnants of this initial cataclysm which is still expanding (expanding universe) having a mean density ~ $10^{-27}$ kg/m$^3$. Discovery of the remnants of the primordial radiation by Arno Penzias and Robert Wilson (1964-65) have given strong support to the Big Bang theory.

According to the above theory it is the fusion of two deuterons which produce the $^4\text{He}$ nuclei. These are the ones which constitute the bulk of the helium present in the universe. The theory also gives the correct overall shape of the abundance curve for nuclei with $A$ upto 100.

Thus we may conclude by saying that the nuclei of the elements are synthesized within the hot interior of the stars, the heavier ones being produced by supernova explosion.
Peaceful Use of Nuclear Energy

15.1 Neutron multiplication and fission chain reaction

The atomic nucleus as a reservoir of huge quantity of energy has been known since the discovery of radioactivity. However, the practical utilization of this energy became possible only after the discovery of nuclear fission. The amount of energy released during radioactive disintegration is not much. Besides, the energy is released at a very slow rate. If we calculate the energy released per second due to the α-disintegration of 1 g of $^{226}$Ra ($Q_\alpha = 4.88$ MeV), we get

$$E = \frac{4.88 \times 1.6 \times 10^{-13} \times 3.7 \times 10^{10}}{10^3 \times 3600} = 8 \times 10^{-9} \text{ kWh}$$

This is obviously too small to be of any use for practical purposes.

Energy is also released in many exoergic reactions. For example, in the $^6\text{Li} (d,\alpha) ^4\text{He}$ reaction, $Q = 22.4/8$ or 2.8 MeV per nucleon. Though this is much larger than the energy released per nucleon in uranium fission ($\sim 0.35$ MeV per nucleon) the total energy which is released at a time in such a reaction is again an insignificant quantity. This is because the number of deuterons which causes the reactions to take place is a very small fraction of the total number present in the incident deuteron beam. Further, the energy required for running the accelerator for accelerating the deuteron beam is quite large, much larger than the energy produced in the reactions. Hence it is not possible to utilize the energy released in this way for practical purpose.

During nuclear fission, on the other hand, not only is a large quantity of energy released, but a number of neutrons are also emitted some of which can be utilized for causing fission of other nuclei. If this happens, then more neutrons will become available after the second generation of fission reactions. These can be further utilized to produce fission in the third generation; and so on. Thus the fission reactions, under favorable conditions, can be made to go on in a chain, without the intervention of any external agent, which means that the chain reaction can go on as a

Reference


Problems

1. Calculate the ratio of the A-values of the two peaks in the energy distribution curves of the fission fragments in the fission of $^{235}\text{U}$ by thermal neutrons (see Fig 14.3) and compare it with the ratio of the corresponding A-values of the fission yield vs. A graph (see Fig. 14.2).
2. In a calorimetric experiment the rate of heat generation in a 13.36 g of natural uranium sample undergoing thermal neutron fission was found to be $9.56 \times 10^{13}$ cal/s. In an auxiliary experiment a 54 microgram sample of natural uranium was found to undergo 340 fissions per min by the same thermal neutron flux. What is the energy release per fission?
3. Calculate the energy release in the symmetric fission of the nuclei with the following values of A and Z:
   - A = 238, Z = 92; A = 200, Z = 80; A = 160, Z = 64; A = 120, Z = 50; A = 100, Z = 44; A = 80, Z = 36. Assume $a_2 = 0.019114$ u and $a_3 = 0.0007626$ u.
4. A nucleus with A = 239 and Z = 92 is deformed from the spherical shape such that the deformation parameter $\alpha_2 = 0.1$ while $\alpha_3 = \alpha_4 = 0$. Calculate the percentages in the surface and Coulomb energy changes of the nucleus from those of the spherical shape.
5. In a fusion reactor six deuterons fuse to produce two $^4\text{He}$ nuclei, two protons and two neutrons according to Eqs. 14.12-1 to 14.12-3. The total energy release in these processes is 43.2 MeV. Write down the sequence of equations governing the above processes. Calculate the energy liberated per gram of deuterium in kilowatt-hours.
6. Show that the term in $P_I (\cos \theta)$ in the expression for $r(\theta)$ in Eq. (14.7-7) leads to a bodily translation.