Gamma Rays

6.1 Nature of \(\gamma\)-rays

We have seen that a nucleus may be left in an excited state following \(\alpha\) or \(\beta\) disintegration. The excited nucleus then makes transition to a lower energy state by the emission of electro-magnetic radiation, known as \(\gamma\)-rays. Gamma rays are much more penetrating than the \(\alpha\) or \(\beta\) rays (see § 3.1). They are not deflected by electric or magnetic fields. The French scientist P.V. Villard (1900) was the first to realize that these rays were different from \(\alpha\) or \(\beta\)-rays by observing their great penetrating power.

Emission of \(\gamma\)-rays is analogous to the emission of electromagnetic radiation from the excited states of atoms. In the latter case, the radiation is emitted due to transition of an orbital electron from a higher to a lower energy state and lies in the visible, ultraviolet or infra-red regions. The transition of a valence electron in the atom from one energy state to another usually results in the emission of photons of only a few electron-volts energy. In some cases, this energy may even be a fraction of an electron-volt. On the other hand, the measurement of the energies of the \(\alpha\) or \(\beta\) rays show that these have energies in the million \((10^6)\) electron-volt region. The transitions between the nuclear energy states thus result in the emission of photons with energies ranging from a few thousand electron-volts to several million electron-volts. It was seen in Ch. VIII of Vol. I that the characteristic X-rays emitted due to the transitions of the electrons in the inner shells (K, L etc. shells) of the atoms have energies up to several thousand electron-volts. Evidently, the \(\gamma\)-rays have generally much higher energies than these characteristic X-rays. The characteristic X-rays have wavelengths of the order of an angstrom or even a fraction of an angstrom \((10^{-10}\text{ m})\). The wavelengths of the \(\gamma\)-rays are obviously even shorter than these in many cases.

Using the relation given in § 5.6 in Vol. I, we have

\[
E = h \nu = \frac{hc}{\lambda} = 12412.5\text{ eV} \quad \text{(6.1-1)}
\]

So the wavelengths of the \(\gamma\)-rays of energies \(10^4\), \(10^5\) and \(10^6\text{ eV}\) are respectively, \(1.24\ \text{Å},\ 0.124\ \text{Å},\ \text{and} \ 0.0124\ \text{Å}\). Amongst the naturally radioactive elements, ThC\(^{4}\) emits \(\gamma\)-rays of the shortest wavelength \((0.00474\ \text{Å})\). We get the corresponding \(\gamma\)-energy of ThC\(^{4}\) as \(E_\gamma = 2.62\text{ MeV}\).

In the case of artificial transmutation of elements, \(\gamma\)-rays are observed having energies of 8 to 10 MeV or even higher. The wavelengths of these \(\gamma\)-ray photons are of the order of one thousandth of an angstrom or even shorter.

6.2 Passage of \(\gamma\)-rays through matter

\(\gamma\)-rays are electromagnetic radiation of very short wavelength \((\lambda\ \text{small compared to the interatomic distance } \varphi)\). While passing through matter, \(\gamma\)-ray photons are either completely absorbed or are deflected (scattered) from their path, usually at large angles. For both these reasons, the intensity of a collimated beam of \(\gamma\)-rays is reduced as it passes through matter.

Let \(I\) be the intensity of a collimated beam of monochromatic \(\gamma\)-rays falling on a slab of material of thickness \(dx\) as shown in Fig. 6.1. The diminution in intensity \(dl\) is found to be proportional to the initial intensity \(I\) and to \(dx\), so that we can write

\[
dl = -\mu_I dx \quad \text{(6.2-1)}
\]

On integration, we get

\[
I = I_0 \exp(-\mu x) \quad \text{(6.2-2)}
\]

\(\mu\) is called the attenuation coefficient. If \(x\) has the dimension of length \((m)\) then \(\mu\) has the dimension of the reciprocal of length \((m^{-1})\).

The mass attenuation coefficient is defined as \(\mu_m = \mu/\rho\) where \(\rho\) is the density of the material. We can rewrite Eq. (6.2-2) as

\[
l = l_0 \exp\left(-\frac{\mu}{\rho} x\right) = l_0 \exp\left(-\mu_m \rho x\right) \quad \text{(6.2-3)}
\]

where the thickness of the absorber \((xp)\) is measured in terms of the mass per unit area (kilogram per metre\(^2\)). Then \(\mu_m\) has dimension of \(m^2/\text{kg}\).

If \(\mu_{ab}\) is the absorption coefficient and \(\mu_{sc}\) is the scattering coefficient we can write

\[
\mu = \mu_{ab} + \mu_{sc}
\]

The corresponding mass absorption and mass scattering coefficients are obtained by dividing \(\mu_{ab}\) and \(\mu_{sc}\) respectively by \(\rho\).

If \(n\) be the number of atomic centres producing attenuation per unit volume we can write

\[
n = \frac{N_0 \rho}{M}
\]
Nuclear Physics

where \( N_0 \) is the Avogadro number and \( M \) is the atomic weight. If we write

\[
\sigma = \frac{\mu}{n} = \frac{\mu M}{N_0 \rho}
\]

or,

\[
\mu = \sigma n = \frac{\sigma N_0 \rho}{M}
\]

then since \( n \) has the dimension of the reciprocal of volume \((m^{-3})\), \( \sigma \) has the dimension of an area \((m^2)\). We call \( \sigma \) the cross section for attenuation of the \( \gamma \)-rays. It has two parts:

\[
\sigma = \sigma_{ab} + \sigma_{sc}
\]

where \( \sigma_{ab} = \mu_{ab} / n \) and \( \sigma_{sc} = \mu_{sc} / n \).

\( \sigma_{ab} \) is the cross section for absorption and \( \sigma_{sc} \) is the cross section for scattering.

Theories have been developed for the calculation of the cross-sections of the different processes by which \( \gamma \)-rays interact with matter.

Experimentally one can measure the attenuation coefficient \( \mu \) by plotting \( \ln I \) as a function of the thickness \( x \) of the material traversed. The graph should be a straight line of slope \((-\mu)\). Eq. (6.2-2) gives

\[
\ln I = \ln I_0 - \mu x
\]

If the thickness required to reduce the intensity of half the initial value is \( d_{1/2} \), then we can write

\[
\ln \frac{I}{I_0} = \ln \frac{1}{2} = -\mu d_{1/2}
\]

or,

\[
\mu = \ln 2 \cdot \frac{d_{1/2}}{d_{1/2}} = 0.693
\]

\( d_{1/2} \) is known as the half-value thickness.

The principal processes by which \( \gamma \)-rays interact with matter are
(i) Photoelectric effect; (ii) Compton scattering and
(iii) Electron-positron pair-production. We can then write the total cross-sections for the above three processes:

\[
\sigma = \sigma_{ph} + \sigma_{c} + \sigma_{p}
\]

Here \( \sigma_{ph}, \sigma_{c}, \sigma_{p} \) are respectively the cross sections for photoelectric absorption, Compton scattering and pair-production.

In the first two processes, the \( \gamma \)-photons collide with the atomic electrons, while in the third, mainly with the nuclei. At lower energies, the first two processes are important; Pair production becomes important at higher energies \((E_{\gamma} > 3 \text{ MeV})\). We shall now discuss the three processes one by one. It must be stated that the theories of all these processes are based on quantum electrodynamics and are beyond the scope of the present book. We shall discuss only the important results.

6.3 Photoelectric absorption of \( \gamma \)-rays

If a photon of energy \( h\nu \) greater than the energy of binding \( B_x \) of an electron in an atomic shell, is incident on the latter, then the electron absorbs the whole quantum of energy of the photon and is emitted from the atom with a kinetic energy

\[
E = h\nu - B_x
\]

These electrons are known as photoelectrons (see Ch V, Vol. I). Since the \( \gamma \)-energy is usually quite high, photoelectrons can be emitted from the inner electronic shells \((K, L \text{ etc.})\). The photoelectrons emitted from a particular shell have the same kinetic energy for a given \( \gamma \)-energy. If the electrons in the \( K, L, M \text{ etc.} \) shells have the binding energies \( B_{K}, B_{L}, B_{M} \text{ etc.} \), we can write

\[
E_K = h\nu - B_K, \quad E_L = h\nu - B_L, \quad E_M = h\nu - B_M \quad \cdots \quad (6.3-2)
\]

so that

\[
h\nu = E_K + B_K = E_L + B_L = E_M + B_M \quad \cdots \quad (6.3-2a)
\]

Since

\[ B_{K} > B_{L} > B_{M} \cdots \]

it follows that \( E_K < E_L < E_M < \cdots \). If the energies of the photoelectrons are measured by a magnetic \( B \)-spectrometer, we get a number of peaks at certain definite values of the magnetic field. Higher the energy of the electrons, greater is the magnetic field at which the corresponding peak is produced. This means that \( K \)-photopeak is produced at a lower magnetic field than the \( L \)-peak, which is produced at a lower field than the \( M \)-peak, and so on.

It may be noted that photoelectric absorption can take place from an electron bound in the atom. A free electron cannot absorb the entire energy of a photon, since energy and momentum conservation cannot be satisfied simultaneously in this case. The more tightly bound the electron is in an atomic shell, more probable is photoelectric emission. Thus the emission of the \( K \)-shell electrons is the most probable (about 80% of the time). The variation of the photoelectric absorption coefficient as a function of the photon energy shows sharp discontinuities at the energies equal to the minimum energies required to eject the electrons bound in the \( K, L, M \text{ etc.} \) shells (absorption edges). As the energy \( h\nu \) increases beyond the \( K \)-absorption edge, which appears at a higher energy than the other peaks \((L, M, \text{ etc.})\), there is a rapid decrease in the value of \( (\mu_{ph} \cdot \lambda) \). Thus the photoelectric effect is important at relatively low photon energies (see Fig. 6.2).

The total photoelectric absorption cross-section for \( K \)-electrons is given by

\[
\sigma_{ph} = \sigma_{e} Z^{3} \alpha^{4} 4\sqrt{2} \left( \frac{m_0 c^2}{h \omega} \right)^{7/2}
\]

where \( \sigma_{e} = 8\pi r_0^2 / 3 \) is the Thomson scattering cross-section;

\[ r_0 = \frac{e^2}{4\pi\epsilon_0 m_0 c^2} = 2.81 \times 10^{-13} \text{ m} \]

is the classical electron radius. The above equation is valid for the non-relativistic region \((h \omega < m_0 c^2)\). \( \alpha \) is Sommerfeld's fine structure constant and \( Z \) is the atomic number of the

absorber. The above equation shows that $\sigma_{ph}$ falls off rapidly with increasing $\gamma$-energy $E_\gamma$ at lower energy.

Since the wavelength $\lambda = h/\omega$, we get

$$\sigma_{ph} \propto \lambda^{-2}$$  \hspace{1cm} (6.3-4)

In the relativistic region ($\omega > m_0 c^2$), the photoelectric absorption cross-section for the K-shell electrons becomes

$$\sigma_{ph} = \frac{3}{2} Z^2 \alpha^2 \left( \frac{m_0 c^2}{\omega} \right)$$  \hspace{1cm} (6.3-5)

Thus at high $\gamma$-energy $\sigma_{ph}$ falls off more slowly with $E_\gamma$. Obviously in this case

$$\sigma_{ph} \propto \lambda^{-2}$$  \hspace{1cm} (6.3-6)

Both Eqs. (6.3-3) and (6.3-5) show that high Z absorbers (e.g., lead with $Z = 82$) are more effective for photoelectric emission than low Z absorbers (e.g., aluminium with $Z = 13$).

In the neighbourhood of the absorption edges Eq. (6.3-3) does not hold. In this case the energy of the ejected electron $E_e = h \omega - B_e$ is comparable to the electron binding energy $B_e$ in the atomic shell. For the K-shell, one gets per electron

$$\sigma_{ph} = 128 \frac{\pi}{3} \frac{e^2}{m_0 c^2} \sqrt{\frac{B_e}{\omega}} \exp(-4 \xi^2)$$  \hspace{1cm} (6.3-7)

where

$$\xi^2 = \frac{B_e}{h \omega - B_e} = \frac{(Z e^2)^2}{\hbar \nu - B_e}$$

$\xi^2$ is the ratio of the K-shell binding energy of the electron to the kinetic energy of the ejected electron.

Here we have put the K-shell binding energy for the electron as $B_e = \hbar \omega - E_e$ so that $\nu_e$ is the frequency of the K-absorption edge.

The differential cross section for photoelectron emission for $E_\gamma < m_0 c^2$ is given by

$$d\sigma_{ph} = 4 \sqrt{2} Z^2 \alpha^2 \left( \frac{m_0 c^2}{E_\gamma} \right)^{7/2} \times \frac{e^2 \sin^2 \phi \cos^2 \phi}{\left( 1 - B \cos \theta \right)^4} d\Omega$$  \hspace{1cm} (6.3-8)

The above equation gives the probability of photoelectron emission into the solid angle $d\Omega$; $\theta$ is the angle between the $\gamma$-momentum $p_\gamma$ and the emitted electron momentum $p_e$; $\phi$ is the angle between the planes $(p_e, p_\gamma)$ and $(X, p_\gamma)$ where $X$ is the electric vector of the incident $\gamma$-ray.

The above formula shows that the photoelectrons emitted by low energy $\gamma$-rays are predominantly in the direction of the electric vector $X$ of the incident radiation ($\phi = 0$). In the direction of the incident light quantum ($\theta = 0$), the intensity of photoelectron emission is zero.

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**Gamma Rays**

As the energy $E_\gamma$ increases, the electron intensity distribution is shifted more in the forward direction.

### 6.4 Compton scattering of $\gamma$-rays

Compton scattering of $X$-rays was discussed in detail in Ch. VIII of Vol. I. When a photon of energy $h\nu$ suffers Compton scattering at the angle $\theta$ from an electron, regarded free, its wavelength $\lambda$ is increased by an amount

$$\Delta \lambda = 2 \lambda \sin^2 \theta/2$$

where $\lambda = h/m_0 c = 2.4263 \times 10^{-12}$ m is a universal constant, known as the Compton wavelength. $h$, $m_0$, and $c$ are respectively the Planck's constant, electron rest mass and velocity of light in vacuum. If $v'$ be the frequency of the scattered photon, then the kinetic energy of the recoil electron is given by

$$E_e = E_\gamma - E'_\gamma = h\nu - h\nu'$$  \hspace{1cm} (6.4-1)

The recoil energy $E_e$ depends on the direction of recoil. If $\phi$ is the angle of recoil, then it can be shown that

$$\cot \phi = (1 + \alpha) \tan \theta/2$$  \hspace{1cm} (6.4-2)

Here $\alpha = h/m_0 c^2$. Then the kinetic energy of the recoil electron becomes

$$E_e = \frac{2 \alpha h \nu \cos^2 \phi}{(1 + \alpha)^2 - 4 \alpha \cos^2 \phi}$$  \hspace{1cm} (6.4-3)

For the forward scattering of the photon ($\theta = 0$), the electron recoils in the direction $\phi = \pi/2$. For the backward scattering of the photon ($\theta = \pi$), the recoil electron is emitted in the direction $\phi = 0$. Thus though the photon can be scattered in all directions $\theta$ from 0 to $\pi$, the recoil electrons can be emitted in the forward directions only from $\theta = 0$ to $\theta = \pi/2$. The maximum energy of the recoil electron occurs for $\phi = 0$ (i.e., for backward scattering of the photon). It is given by

$$E_{e(\max)} = \frac{2 \alpha h \nu}{1 + 2 \alpha}$$  \hspace{1cm} (6.4-4)

As $\alpha$ becomes very large (i.e., $h \nu > m_0 c^2$), $E_{e(\max)} \rightarrow h\nu$.

O. Klein and Y. Nishina were the first to develop the theory for calculating the cross-section of Compton scattering on the basis of quantum electrodynamics.

The Compton scattering cross-section per electron is related to the mass scattering coefficient ($\mu_{\gamma}$)$_m$ as follows:

$$\mu_{\gamma} = \frac{m_e}{m_0} \frac{\sigma_e}{\rho} = \frac{N_e Z \sigma_e}{M}$$  \hspace{1cm} (6.4-5)

$n_e = (N_e \rho Z)/M$ is the number of electrons per unit volume of the scatterer, $Z$ being the atomic number of the scatterer.

From the Klein-Nishina formula, the following asymptotic expressions are obtained.*

* See ibid.
For low energies ($\alpha << 1$): $$\sigma_c = \sigma_e \left( 1 - 2\alpha + \frac{26}{5} \alpha^2 + \ldots \right) \ldots (6.4-6a)$$

For high energies ($\alpha >> 1$): $$\sigma_c = \frac{3\sigma_e}{8\alpha} \left( \ln 2\alpha + \frac{1}{2} \right) \ldots (6.4-6b)$$

The above asymptotic expressions for $\sigma_c$ show that at low energies, $\sigma_c$ decreases slowly with increasing photon energy $h\nu$, while at higher energies it falls off more rapidly (roughly as $1/\nu^2$). From Eq. (6.4-5) we thus see that mass Compton scattering coefficient ($\mu_c$) has the same type of variation with the photon energy. Further, since $\sigma_c$ represents scattering cross-section per electron, it is obviously independent of $Z$. Eq. (6.4-5) then shows that ($\mu_c$) is almost the same for all elements, since $Z/M$ does not change much for the different elements. So Compton scattering probability is more or less the same for the different elements.

![Graph](image)

**Fig. 6.2.** Mass absorption coefficient for lead ($Z = 82$) as function of $\gamma$-energy.

In Fig. 6.2 is plotted the variation of ($\mu_c$) with the energy of the $\gamma$-ray for lead.

### 6.5 Electron-positron pair production by $\gamma$-rays

This is the third important process by which $\gamma$-rays interact with matter. We have seen in § 5.5 that some artificially produced radioactive nuclei decay by positron emission. The positron is the antiparticle of the electron, having the same mass and spin ($1/2$) as the electron, carrying a charge equal and opposite to that of the electron.

The existence of the positron was predicted by the British scientist P.A.M. Dirac (1928) when he developed the quantum mechanical theory of the relativistic electron. This was before the experimental discovery of the positron. The spin of the electron, which had been introduced earlier by Goudsmit and Uhlenbeck on ad hoc basis to explain the doublet structure of the alkaline spectral lines as well as the anomalous Zeeman effect (see Ch. VI, Vol. I) followed quite naturally from the theory of Dirac.

**Gamma Rays**

**Dirac theory**

Dirac tried to develop a quantum mechanical theory of the electron which took into account the relativistic relation between the energy and momentum; $E^2 = p^2c^2 + m_0^2c^4$ where $E$ is the total energy and $p$ is the momentum of the particle; $m_0$ is the rest mass of the particle. Dirac sought a linear relationship between $E$ and $p$ whose operator representations are:

$$\hat{\rho} = \frac{\hat{x}}{i}, \quad \hat{E} = \hbar \frac{\partial}{\partial t}$$

The equations which Dirac postulated for a free (relativistic) particle has the following form:

$$i\hbar \frac{\partial \psi}{\partial t} = \hbar c \left( \frac{\partial}{\partial x} \psi + \beta mc^2 \psi \right) \ldots (6.5-1)$$

where $m = m_0 \sqrt{1 - \beta^2}$; $m$ is the relativistic mass. The three components of $\alpha$ and $\beta$ anticommute and hence cannot be numbers. They are $4 \times 4$ matrices. So the wavefunction $\psi$ must have four components.*

Dirac interpreted two of the components of $\psi$ to represent the two possible spin states of a spin $1/2$ particle (electron) with spins ‘up’ and ‘down’. The other two solutions, according to Dirac, are the two spin states of another spin-$1/2$ particle, called the antielectron. According to Dirac the two particles would have opposite charges, but the same mass spin, etc.

Thus Dirac’s theory predicted that particles could have an intrinsic spin and would also have antiparticles.

An important conclusion of Dirac electron theory is that the total energy of the electron, inclusive of its rest energy, can be both positive and negative:

$$E = \pm \sqrt{p^2c^2 + m_0^2c^4}$$

This relation shows that the total energy of the electron can be either greater than $+ m_0 c^2$ or less than $- m_0 c^2$ where $m_0$ is the rest mass of the electron. In Fig. 6.3, are shown the total energies possible (shaded region) for an electron according to Dirac theory. Evidently, the electron cannot have any energy between $+ m_0 c^2$ and $- m_0 c^2$.

![Graph](image)

**Fig. 6.3.** Positive and negative energy states of the electron.

* See Vol. I of this book, Appendix A-XI
An electron of positive energy $E > m_0 c^2$ is the electron that is actually found in the physical world. However an electron in a negative energy state does not normally manifest its existence in the physical world. We know that all physical systems have a natural tendency to go down to a state of lower energy from that of higher energy. If the existence of the negative energy state of the electron, as predicted by the Dirac theory, is a physical reality, then all electrons in the positive energy states will have the tendency to go down to the negative states and would thus be lost to the physical world. However, this does not actually happen, since according to Dirac, all the negative energy states are completely filled with electrons. As the electrons obey Pauli's exclusion principle, no two electrons can occupy the same state. So no electron from a positive energy state can make transition to any of the negative energy states, since the latter are all filled with electrons.

However, if by some means an electron is removed from a negative energy state, then a vacancy or a hole is created in the sea of negative energy states filled with electrons. Since such a hole is created due to the absence of a negatively charged electron, it will behave as a positively charged particle with charge $-e$ or $+e$. Further, the absence of a particle in the state of negative energy $-E$ and of positive momentum $p$ will manifest as a particle of positive energy $-E$ or $+E$ and positive momentum $-p$ or $+p$. Thus the hole created in the negative energy state will appear as a positively charged particle in the positive energy state and hence should be observable in our physical world.

At first Dirac had thought that this hypothetical particle was a proton. But this did not seem very plausible, since the proton is so much heavier than the electron.

Subsequently, in 1933, C.D. Anderson discovered a new particle in the cosmic rays, having the same mass as the electron, but carrying a charge equal and opposite to that of the electron, using a Wilson cloud chamber at the California Institute of Technology in the U.S.A. We shall discuss about this experiment in detail in Chapter XIX.

It is said that when Anderson made this very important discovery, he was not aware of Dirac's theoretical prediction. Nevertheless, the discovery constituted a great triumph of Dirac's theory, since it was immediately realized that this was the antiparticle of the electron predicted by Dirac.

To produce a positron, it is necessary to create a vacancy in a negative energy state of the electrons. Since all negative energy states are already filled with electrons, a vacancy in one of these states can be created only if the electron occupying it is transferred to a vacant positive energy state, since Pauli's exclusion principle forbids its transfer to a state of negative energy. The minimum energy needed for such transition is $2m_0 c^2 = 1.022 \text{MeV}$ (see Fig. 6.3). This energy can be given to the electron in the negative energy state by bombarding it with $\gamma$-ray photons of energy $E_\gamma > 2m_0 c^2$. On absorbing the photon, the electron is transferred to a vacant positive energy state and then it behaves like a normally observable electron. The hole created in the negative energy state, of course, behaves like a positron. Thus the action of the $\gamma$-ray of energy greater than $2m_0 c^2$ is to create simultaneously an electron-positron pair.

Electron-positron pair production cannot take place in vacuum, since energy and momentum are not conserved in this case. Pair creation usually occurs in the immediate vicinity of the nucleus of an atom where the Coulomb electric field is very strong. Due to the interaction between this electric field and the photons, the latter sometimes materializes into an electron-positron pair. So pair production is an example of the transformation of energy into matter.

The probability of pair production is expressed in terms of the quantity known as the cross-section for pair production $(\sigma_p)$ which is related to the mass absorption coefficient for pair production $(\mu_p)^m$ as follows:

$$\mu_p = \frac{N_0}{M} \rho \sigma_p \quad \text{(6.5-2)}$$

$\sigma_p$ gives the probability for producing an electron-positron pair due to the interaction of a photon with an atom present per unit area of the absorber. $\sigma_p$ depends on the energy of the $\gamma$-ray and on the atomic number $Z$ of the absorber. For very high energy photons ($h\nu \gg m_0 c^2$), the pair production cross-section is given by

$$\sigma_p = Z^2 \alpha r_0^2 \left( \frac{28}{9} \ln \frac{183}{Z^2/3} - \frac{27}{2} \right) \quad \text{(6.5-3)}$$

Here $\alpha$ is Sommerfeld's fine structure constant. $r_0$ is the classical electron radius. In the relativistic case, $\sigma_p$ becomes constant, independent of photon energy. At other energies $\sigma_p$ increases with energy of the photon. In the table below the values of $\sigma_p$ calculated theoretically are given for a few typical energies:

<table>
<thead>
<tr>
<th>$h\nu/m_0 c^2$</th>
<th>$3$</th>
<th>$4$</th>
<th>$5$</th>
<th>$6$</th>
<th>$10$</th>
<th>$20$</th>
<th>$50$</th>
<th>$100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_p/Z^2 \alpha r_0^2$</td>
<td>0.085</td>
<td>0.32</td>
<td>0.61</td>
<td>0.89</td>
<td>1.94</td>
<td>3.75</td>
<td>6.4</td>
<td>7.9</td>
</tr>
</tbody>
</table>

$\sigma_p/Z^2 \alpha r_0^2$ is found to be independent of $Z$ except at very high energies. The values given in the above table in the last two cases are for lead.

In Fig. 6.2, shown is the variation of $(\mu_p)^m$ with photon energy. As can be seen, at high energies ($h\nu > 3 \text{MeV}$), pair-production is the most important process for the interaction of $\gamma$-rays with matter.

Since $\sigma_p \propto Z^2$, pair-production probability is much greater in a high $Z$ absorber like lead with $Z = 82$ than in a low $Z$ absorber like aluminium with $Z = 13$.

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* See Quantum Theory of Radiation by W. Heitler.
It may be noted that the electron-positron pair production by photons can also take place in the electric field of an electron. However, the probability of the process is much smaller than in the case of a nucleus. For the electron Z = 1 and hence $\sigma_{\text{e}}$ given by Eq. (6.5-3) is lower by a factor $Z^2$. Even when all the Z electrons in an atom are considered, the probability of pair-production due to the electrons is still much lower than due to the nuclear electric field.

### 6.6 Electron-positron annihilation

The opposite of the electron-positron pair-creation is the phenomenon of electron-positron annihilation. After a positron is created, it loses energy by ionization and other processes while travelling through matter in the same way as an electron does. Finally, when it loses all its energy, it interacts with atomic electron within the substance, which may be regarded to be at rest. As a result of this, both the electron and the positron are completely destroyed and their total mass energy $2m_e c^2$ is converted into two $\gamma$-photons. Since the electron and the positron are at rest at the time of their annihilation, their total momentum, i.e., the initial momentum of the process is zero. So according to the law of conservation of momentum, the final momentum of the photons must also be zero. Hence the two photons must have the same energy $m_\gamma c^2 = 0.511$ MeV each and they must move away in opposite directions with equal and opposite momenta. The photons thus produced are known as annihilation radiation.

The annihilation of the electron and positron can be visualized in the following manner on the basis of Dirac theory. As the positron loses energy, the hole in the negative energy state comes more and more upwards. Finally, when it loses its kinetic energy completely, it comes up to the level $(-m_e c^2)$. At this moment an electron at rest, which has a total energy of $+m_e c^2$ makes a downward transition and fills up the hole in the negative energy state $-m_e c^2$. As a result, an electron is lost from the positive energy state and at the same time, the hole in the negative energy state (i.e., the positron) is also eliminated. Due to this transition, the total change in the energy of the electron, which is $m_e c^2 - (-m_e c^2)$ or $2m_e c^2$ manifests itself as two $\gamma$-ray photons (see Fig. 6.3).

**Positronium**

The mechanism of the $e^-e^+$ annihilation process actually involves the formation of a short lived hydrogen-like atomic system, with the electron and positron revolving round their common centre of mass, known as the positronium. The properties of the positronium can be studied during the life time of its ground state, which is of the order of $10^{-8}$ s. The reduced mass of the positronium is almost half that of the hydrogen atom: $\mu_{\text{e}} = m_e^2/2m_e = m_e/2$. So its Rydberg constant and binding energy are half those of the hydrogen atom, while its radius is twice as large.

### Gamma Rays

The resultant spin S of the positronium can either be 0 or 1, depending on whether the spins of the electron and positron are antiparallel or parallel. These are known as para-positronium and ortho-positronium respectively. The total angular momentum $J = L + S$, where the orbital angular momentum $L = 0, 1, 2, 3, \ldots$. Thus the states of the ortho-positronium are $^1S_0, {}^3P_0, {}^3P_1, {}^3P_2, {}^3D_1, {}^3D_2, {}^3D_3$, etc. while those of para-positronium are $^1S_0, {}^1P_1, {}^1D_2$ etc.

The annihilation of the positronium takes place from the S-state. Ortho-positronium annihilates into three $\gamma$-quanta while para-positronium annihilates into two $\gamma$-quanta. The former is less probable than the latter, the respective mean lives in the two cases being $\tau$ (ortho) = $1.4 \times 10^{-7}$ s and $\tau$ (para) = $1.25 \times 10^{-10}$ s.

### 6.7 Determination of $\gamma$-Ray Energy

The energy of $\gamma$-rays can be determined either by measuring its wavelength or by measuring the energy of the electron ejected or created by it during its interaction with matter.

**a) Wavelength measurement**

When the $\gamma$-rays have wavelengths comparable to the X-rays, crystal diffraction method can be used for measuring their wavelengths. For example, rotation photograph method described in Ch. XVIII, Vol. I, has been used to measure the minimum wavelength of 0.016 Å of the RaC$^+$ $\gamma$-rays.

The curved crystal method of J.W.M. Du Mond and Y. Cauchois has been used to determine wavelengths less than 0.01 Å fairly accurately (see Ch. VIII, Vol. I).

By measuring the wavelength $\lambda$, it is possible to determine the photon energy by using Eq. (6.1-1).

**b) Photoelectric method**

It is possible to measure the kinetic energy of the photoelectrons ejected by the $\gamma$-rays by means of a magnetic spectrometer described in Ch. V. A foil of some high Z material, e.g., gold, used as a radiator, is placed near the radioactive substance emitting $\gamma$-rays. Photoelectrons emitted from the foil by the $\gamma$-rays are focussed into a Geiger-Müller counter placed in the magnetic spectrometer. A number of photo-peaks is produced by the electrons ejected from the different shells by the $\gamma$-rays of a particular energy.

A few such peaks are shown in Fig. 6.4. From the Br values corresponding to the positions of the peaks, it is possible to determine the kinetic energies of the photoelectrons ejected from the different shells. Using Eq. (6.3-2a), it is then possible to determine the energy of the $\gamma$-ray photon $h\nu$ independently from these measurements for the electrons from the different shells. In this way very accurate determination of the $\gamma$-energy is possible.
\[ E_\gamma = W_e + W_p = \sqrt{p_e^2 c^2 + m_0^2 c^4} + \sqrt{p_p^2 c^2 + m_0^2 c^4} \]

where the p's are the momenta of the particles. If B be the magnetic induction field, we have \( p_x = Be_x \) and \( p_p = Be_p \), where \( e_x \) and \( r_p \) are the radii of curvature of the electron and the positron respectively. For \( pc >> m_0 c^2 \) for both the particles, we then get

\[ E_\gamma = (p_e + p_p) c = B_e c (e_x + r_p) \]

which proves the statement made above.

It is possible to determine \( \gamma \)-energies up to about 100 MeV by this method.

(e) Conversion electron method:

We have mentioned about the appearance of sharp peaks of internal conversion electrons superimposed upon the \( \beta \)-spectrum in certain cases. Their origin will be explained in § 6.11. As will be seen, an excited nucleus of energy \( E_i \) instead of going down to a lower state of energy \( E_f \) by the emission of \( \gamma \)-ray of energy \( hv = E_i - E_f \), sometimes makes a transition to the lower state by the emission of an electron from one of the atomic shells of the same atom. The energy of this conversion electron is

\[ E_e = E_i - E_f - B_e = hv - B_e \]

where \( B_e \) is the binding energy of the electron in the orbit. If the conversion electron is emitted from the K-shell we can write

\[ E_{ek} = E_i - E_f - B_{ek} = hv - B_{ek} \]

\[ hv = E_{ek} + B_{ek} \]

If the energy of the conversion electron is measured by a magnetic spectrometer, then it is possible to determine the \( \gamma \)-ray energy accurately.

(f) Scintillation spectrometer method:

This method gives \( \gamma \)-energy very accurately. We have seen that Rutherford and others used a scintillating screen (e.g., ZnS) in front of the objective of a microscope and detected the incidence of the \( \alpha \)-particles on the screen by observing the scintillations produced on the latter by the \( \alpha \)-particles.

The method was greatly improved in later years by coupling a scintillating crystal, called a phosphor, with a photomultiplier tube and detecting the electrical pulses obtained from the latter electronically. The details of the method will be discussed in Ch. VII.

\( \gamma \)-rays travelling through a thallium-activated sodium iodide crystal NaI(Tl) eject photoelectrons, Compton recoil electrons or electron-positron pairs from the crystal. These electrons, by their interaction with the atoms or molecules of the crystal, generate a very
large number of visible or ultraviolet photons within a very short interval of time, which appear as a sudden flash of light or scintillation. The number of these visible or ultraviolet photons and hence the intensity of the scintillation, is linearly dependent on the energy of the incident γ-ray photon. The light from the scintillation falls on the cathode of the photomultiplier tube and emits electrons from the latter. The number of electrons is then greatly multiplied (by a factor of about $10^6$ or more) by the action of the photomultiplier. As a result, the momentary electron current produced in the tube generates a very fast electrical pulse at the anode, with an amplitude proportional to the energy of the γ-ray passing through the phosphor.

The electrical pulses from the photomultiplier are further amplified and recorded electronically after they are sorted out with the help of a pulse-height selector. By counting electronically the number of electrical pulses of different amplitudes, a pulse-height spectrum is obtained, which has the appearance shown in Fig. 6.6. It can be seen from the figure that a sharp peak is produced in the pulse height spectrum at a particular value of pulse height (channel number). This peak is mainly due to the photoelectrons produced by the γ-rays within the phosphor.

If the instrument is calibrated by means of γ-rays of known energy, then the unknown energy of other γ-rays can be determined.

It should be noted that though the energy of the photoelectrons $(E_γ - B_e)$ is less than the γ-energy, the photo-peak in the pulse-height spectrum appears at the full γ-energy $E_γ$. As the photoelectron is emitted, say from the K-shell, an electron from an outer shell (say the L-shell) makes a transition to the K-shell, emitting the characteristic K X-ray photon, which is also absorbed within the phosphor and contributes to the number of visible and ultraviolet photons constituting the scintillation. The vacancy in the L-shell is similarly filled up by transition from an outer shell and the corresponding characteristic X-ray emitted is also absorbed in the phosphor. In this way the entire deficit of energy $(B_e)$ of the photoelectron is compensated due to the absorption of all the X-ray photons in the phosphor, following the photoelectron emission.

It is to be noted further that there are some contributions from the Compton electrons as also from the electron-positron pairs in producing the photopeak. If the Compton scattered photon is absorbed within the phosphor along with the recoil electron, their combined energy becomes equal to the energy of the incident γ-ray. So the net effect is for them to produce jointly an electrical pulse of the same amplitude as that due to the photoelectron.

In the case of the electron-positron pair though the sum of their kinetic energies is less than $E_γ$, by the amount $2m_e c^2$, the positron is finally annihilated after it is brought to rest within the phosphor by ionization and other processes. The total energy of the two photons produced due to the annihilation is $2m_e c^2$. If these are both absorbed within the phosphor, they make up for the deficit of $2m_e c^2$ from the γ-energy mentioned above, so that the net effect is to produce a pulse of the same amplitude as the photopeak, as in the case of the Compton electrons.

If the Compton scattered photon is not absorbed but comes out of the phosphor, then the electrons recoiling in different directions produce pulses of different amplitudes because of the continuous distribution of their energies. This gives rise to a continuous background in the pulse-height spectrum, as shown in Fig. 6.6a.

On the other hand, if the two electron-positron annihilation photons, each of energy $m_e c^2$, escape from the phosphor, then an additional peak is produced at the pulse height corresponding to the energy $E_γ - 2m_e c^2$. If however, only one of these photons escapes from the phosphor, then a peak at the pulse height corresponding to the energy $E_γ - m_e c^2$ will be produced. All these peaks can be seen in Fig. 6.6b in which the three peaks correspond to the full γ-ray energy $E_γ = 4.43$ MeV, $E_γ - m_e c^2 = 3.92$ MeV and $E_γ - 2m_e c^2 = 3.41$ MeV respectively.

\[ 137\text{Cs} \]
\[ 3\text{in} \times 3\text{in} \]
\[ \text{NaI(Tl)} \]
\[ 0.662\text{MeV} \]
\[ \text{Backscatter peak} \]
\[ \text{Compton distribution} \]
\[ \text{Photopake} \]

(a) \[ \text{Counting rate} \]
\[ \text{Pulse height} \]

(b) \[ \text{Counting rate} \]
\[ \text{Pulse height} \]

Fig. 6.6 Pulse height spectrum for γ-rays by scintillation spectrometer. (a) For $E_γ < 2m_0 c^2$; (b) For $E_γ > 2m_0 c^2$. 

\[ E_γ < 2m_0 c^2; \quad (b) \text{For } E_γ > 2m_0 c^2. \]
For further details about the scintillation counter, see § 7.8. Other methods of γ-energy measurement are discussed in Ch. VII.

6.8 γ-ray spectra and nuclear energy levels

We have seen γ-rays are usually emitted due to transitions from an excited nucleus formed after α or β disintegration to a lower energy state. Sometimes, a particular radioactive element emits α-particles of more than one energy, due to which the daughter nucleus is left in different discrete energy levels. There should be correlation between the energy differences of the different α-groups and the energies of the γ-rays emitted due to transitions between the levels of the daughter nucleus so formed. Similarly there are many nuclei which emit β-ray groups of different maximum energies. There should be definite correlations between the differences in the maximum energies of the different β-groups and the energies of the γ-rays due to transitions between the levels of the daughter nucleus.

As an example we may consider the α-disintegration of $^{226}$Ra nucleus:

$$^{226}\text{Ra} \rightarrow ^{222}\text{Rn}$$

Two different α-ray groups are observed in this decay, having the kinetic energies 4.795 and 4.611 MeV respectively. The daughter nucleus $^{222}$Rn is found to emit γ-rays of energy 0.188 MeV. The α-disintegration energies ($Q_{\alpha}$) in the two cases can be calculated using Eq. (4.5-1) and come out to be 4.881 and 4.694 MeV respectively. The difference between the two $Q_{\alpha}$-values is 0.187 MeV which agrees very well with the γ-energy given above. In Fig. 6.7 is shown the energy level diagram for the above α and γ transitions.

From the figure, it can be seen that the α-decay of $^{226}$Ra leads to the formation of the daughter nucleus $^{222}$Rn either in the ground state or in the excited state of energy 0.188 MeV. In the latter case, transition to ground state gives rise to the emission of the γ-ray of energy 0.188 MeV.

In § 4.11 we discussed about the fine structure of the α-ray spectra. As an example, in the ThC → Th C" α-decay, five different α-groups are observed, having the energies 6.11, 6.07, 5.76, 5.62 and 5.60 MeV respectively. The observed γ-ray energies are 0.491, 0.471, 0.451, 0.431, 0.327, 0.287, 0.162 and 0.040 MeV. The corresponding α and γ transitions are shown in the energy level diagram in Fig. 6.8 from which the origins of the different γ-rays are clear.

It may be noted that the γ-rays following α-decay have rather low energies (<0.5 MeV) because of the strong dependence of the α-emission probability on α-energy (exponential dependence; see § 4.9) which inhibits emission of very low energy α-particles.

---

\[ \text{Fig. 6.8. γ-transitions between the energy levels of ThC'' (}^{212}\text{Bi})\text{ formed by the α-decay of ThC (}^{212}\text{Bi}). The α-energies shown are the disintegration energies } Q_{\alpha}. \]

\[ \text{Fig. 6.9. γ-transitions in } ^{234}\text{Pa}, ^{20}\text{Ne}, ^{34}\text{S}, \text{and } ^{64}\text{Ni formed by β-decay of the respective parent nuclei UX (}^{234}\text{Th), }^{20}\text{F}, ^{34}\text{Cl}, \text{and } ^{64}\text{Cu}. \]
Again in the decay of $^{234}$Th, to $^{234}$Pa by $\beta$ emission, $\beta$-rays of two different maximum energies $0.020$ and $0.11$ MeV are observed. The difference between these two maximum energies is $0.09$ MeV. It is found that the daughter nucleus $^{234}$Pa emits $\gamma$-rays of energy $0.09$ MeV. The corresponding energy level diagram is shown in Fig. 6.9a.

A few other energy level diagram showing the $\beta$ and $\gamma$ transitions of some other nuclei are also shown in Fig. 6.9.

Unlike the case of nuclei formed by $\alpha$-decay those formed as a result of $\beta$-decay, are found to emit fairly high energy $\gamma$-rays because of the relatively less sensitive dependence of the $\beta$-decay probability on $\beta$-energy.

### 6.9 Radiative transitions in nuclei

When a nucleus makes a transition from an excited state to a state of lower energy, it usually emits electromagnetic radiation ($\gamma$-ray) without change of $A$ or $Z$. This is known as radiative transition. Because such a transition is an electromagnetic phenomenon, it is theoretically better understood than the $\alpha$ or $\beta$ transition.

The transition probability from a level is related to the width of the level. According to Heisenberg’s uncertainty principle (see § 9.14, Vol. I), there is a limit to the sharpness of the energy of a quantum system which is related to its mean life time given by

$$\Delta E \Delta t \geq \hbar$$

where $\hbar = h/2\pi$, $h$ being Planck’s constant. Thus, unless the system has an infinite life-time its energy will always have an uncertainty $\Delta E = \Gamma$, which is called the width of the level. Greater the width $\Gamma$, shorter is the life-time $\tau$ of the level:

$$\tau = \frac{\hbar}{\Gamma} \quad \ldots(6.9-1)$$

The transition probability $T$ per second which is equal to the decay constant $\lambda$ is

$$T = \lambda = \frac{1}{\tau} = \frac{\Gamma}{\hbar} \quad \ldots(6.9-2)$$

So the width of the level determines the transition probability $T$. Wider the level, more probable is the transition from it (see Fig. 6.10). Because of the finite width of the level, the emitted $\gamma$-ray line will have a finite spread in its frequency (and wavelength).

If an excited level can simultaneously decay by the emission of several types of radiations (e.g. $\alpha$, $\beta$ and $\gamma$), the total transition probability is equal to the sum of the transition probabilities for the emission of the different types of radiations:

$$T_{\text{tot}} = T_{\alpha} + T_{\beta} + T_{\gamma} \quad \ldots(6.9-3)$$

$$\Gamma_{\text{tot}} = \Gamma_{\alpha} + \Gamma_{\beta} + \Gamma_{\gamma} \quad \ldots(6.9-4)$$

where $\Gamma_{\text{tot}}$ is the total width of the level, while $\Gamma_{\alpha}$ etc. are the partial widths for $\alpha$-emission etc. If $\tau_{\alpha}$, $\tau_{\beta}$, $\tau_{\gamma}$ are the mean lives against the emission of the different radiations we get,

$$\frac{1}{\tau} = \frac{1}{\tau_{\alpha}} + \frac{1}{\tau_{\beta}} + \frac{1}{\tau_{\gamma}} \quad \ldots(6.9-5)$$

Here $\tau$ is the resultant mean-life of the level.

If a particular level decays by $\gamma$ emission only, we can write $\tau = \tau_{\gamma}$ and $\Gamma = \Gamma_{\gamma}$. Now radiative width of a level is found to be of the order of 0.1 eV or less. So we get

$$\frac{\tau_{\gamma}}{\Gamma_{\gamma}} \approx \frac{1.05459 \times 10^{-34}}{0.1 \times 1.60219 \times 10^{-19}} \approx 6.58 \times 10^{-15} \text{ s}$$

Thus the life-time against radiative transition is of the order of $10^{-14} \text{ s}$ or more. If radioactive transition is the only possibility, then the ground state usually has an infinite life ($\tau \rightarrow \infty$) so that its width $\Gamma = 0$. Hence the ground state in this case is absolutely sharp. If however, the ground state of the system can decay by particle emission (e.g. in the case of $\alpha$ or $\beta$ decay), then it will not be absolutely sharp, but will have a finite width.

Let us now consider the mechanism of emission of electromagnetic radiation by a system of charges.

It is known that the static potential $V$ at some point $P$ due to a distribution of charges around some arbitrarily chosen origin of coordinates $0$ is given by

$$4\pi \varepsilon_0 V = \sum q_i \frac{1}{r_i} \frac{1}{R_0 - r_i} = \sum q_i \left( \frac{1}{r_i^2} - \frac{1}{R_0^2} \cos \theta_i \right)^{1/2} \quad \ldots(6.9-6)$$

where $R_0$ is the position vector of the field-point $P$ and $r_i$, $\theta_i$ determine the position of the $i$th charge $q_i$ as shown in Fig. 6.11. $\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ is the permittivity of vacuum.

When the charge-distribution is confined within a small region of linear dimension $a << R_0$, then obviously $R_0 >> r_i$ so that the above expression for the potential can be expanded as follows:

$$4\pi \varepsilon_0 V = \sum \frac{q_i}{R_0} + \frac{1}{R_0^2} \sum q_i r_i \cos \theta_i$$

$$+ \frac{1}{3} \sum q_i \left( \frac{3}{2} \cos^2 \theta_i - \frac{1}{2} \right)$$

$$+ \ldots + \sum \frac{1}{R_0^3} \sum q_i r_i \cos \theta_i$$

$$+ \ldots \quad \ldots(6.9-7)$$

Fig. 6.11. Potential due to a charge distribution.
For $L = 0$, $V_0 = \frac{1}{4 \pi \varepsilon_0 R_0^3} \Sigma q_i$.

This is the potential due to a monopole which is nothing but the point charge $\Sigma q_i$ located at the origin.

For $L = 1$, $V_1 = \frac{1}{4 \pi \varepsilon_0 R_0^3} \Sigma q_i r_i \cos \theta_i$.

This is the potential due to an electric dipole located at the origin having the moment $p = \Sigma q_i r_i$.

Similarly for $L = 2$, we get the potential due to an electric quadrupole located at the origin $V_2 = \frac{1}{4 \pi \varepsilon_0 R_0^3} \Sigma q_i r_i^2 \left( \frac{3}{2} \cos^2 \theta_i - \frac{1}{2} \right)$.

In general, $V_L$ represents the potential due to an electric multipole of order $L$ located at the origin given by

$$V_L = \frac{1}{4 \pi \varepsilon_0 R_0^{L+1}} \Sigma q_i r_i^L P_L(\cos \theta_i)$$

where $P_L(\cos \theta_i)$ is the Legendre polynomial of order $L$.

If the charge distribution has time-varying moments, then it will radiate energy, the intensity of the radiation being determined by the Poynting vector. The different terms in the above expansion correspond to the electric vectors of the radiation fields due to different orders of multipoles. The first term due to the electric monopole does not radiate energy. The second term in the expansion corresponds to the radiation field of the electric dipole (E1) with a time-varying moment. The next term corresponds to the radiation field of an electric quadrupole (E2) of time-varying moment. It also represents the radiation field of a magnetic dipole (M1) of time-varying moment. In general $V_L$ corresponds to the radiation field of an electric $2^L$ pole (EL) or of a magnetic $2^{L-1}$ pole $[M(L-1)]$ of time-varying moments. $L$ is the multipolarity of the radiation and is a measure of the number of units of angular momentum transferred in the radiative transition. $L$ is analogous to the orbital angular momentum transferred in the case of particle emission.

The above classification of the radiative processes is based on the conservation of angular momentum and of parity between the radiating system and the radiation field.

If $L$ is the angular momentum carried away by the photon w.r.t. the origin, then we can write $L = I_i - I_f$.

**Gamma Rays**

where $I_i$ and $I_f$ are the angular momenta of the initial and final states respectively. This means that for given $I_i$ and $I_f$, $L$ can have the following values, in accordance with the vector model discussed in Ch. VI, Vol. I:

$$I_i + I_f \geq L \geq |I_i - I_f|$$

If we consider some preferential direction in space (Z-direction) such that the components of $I_i, I_f$ and $L$ along Z are $m_i, m_f$ and $M \leq L$ respectively, then we have additionally

$$M = m_i - m_f$$

The values of the magnetic quantum numbers $m_i, m_f$, and $M$ are determined by the rules of space quantization discussed in Ch. VI, Vol. I. Eqs. (6.9-13) and (6.9-14) are the selection rules which set limits on the possible multipoles of the transition.

The orientations of the vectors $I_i, I_f$, and $L$ according to vector model are shown in Fig. 6.12.

Actually the transitions are also governed by the parity selection rule. All evidences indicate that parity is conserved in electromagnetic processes: For a given multipolarity $L$, the radiation may have both even and odd parity, unlike in the case of particle emission when the parity is determined by the orbital angular momentum.

The parity selection rule states that in electric multipole transition (EL) the parity is even when $L$ is even (E2, E4 etc.) and odd when $L$ is odd (E1, E3 etc.) so that the parity is given by $(−1)^{L}$. For the magnetic multipole transition (ML), the parity is given by $(−1)^{L+1}$ so that the parity is odd for even $L$ (M2, M4 etc.) and even for odd $L$ (M1, M3 etc.).

**Table 6.1**

<table>
<thead>
<tr>
<th>Multipole type</th>
<th>Multipolarity (L)</th>
<th>Parity change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric dipole (E1)</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>Magnetic dipole (M1)</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>Electric quadrupole (E2)</td>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>Magnetic quadrupole (M2)</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>Electric octupole (E3)</td>
<td>3</td>
<td>Yes</td>
</tr>
<tr>
<td>Magnetic octupole (M3)</td>
<td>3</td>
<td>No</td>
</tr>
<tr>
<td>Electric $2^L$ pole (EL)</td>
<td>$L$</td>
<td>$(−1)^L$ (Yes for $L$ odd, No for $L$ even)</td>
</tr>
<tr>
<td>Magnetic $2^L$ pole (ML)</td>
<td>$L$</td>
<td>$(−1)^{L+1}$ (Yes for $L$ even, No for $L$ odd)</td>
</tr>
</tbody>
</table>
An important point to note is that no multipole radiation with $L = 0$ is possible. Hence $I_l = 0$ to $I_f = 0$ transition is completely forbidden. (See however § 6.13.)

In table 6.1 we summarize the angular momentum and parity selection rules for different types of multipole transitions.

In table 6.2 we list the types of radiation allowed in the transitions $I_l \rightarrow I_f$ for different values of $I_l$ and $I_f$ consistent with the above selection rules.

### Table 6.2

<table>
<thead>
<tr>
<th>$I_l$</th>
<th>$I_f$</th>
<th>Possible $L$-values</th>
<th>Parity change</th>
<th>Radiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Yes, No</td>
<td>No radiation</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
<td>Yes</td>
<td>E1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No</td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Yes</td>
<td>M2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No</td>
<td></td>
<td>E2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1, 2, 3</td>
<td>Yes</td>
<td>E1’ (M2, E3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(M1, E2) M3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>2, 3, 4</td>
<td>Yes</td>
<td>(M2, E3) M4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No</td>
<td></td>
<td>E2 (M3, E4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Yes</td>
<td>E1 (M2, E3) M4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No</td>
<td></td>
<td>(M1, E2) M3, E4 M5</td>
</tr>
</tbody>
</table>

It is found that the probability of multipole emission decreases rapidly with increasing $L$. For the same $L$, emission of the electric radiation is much more probable than the emission of the magnetic radiation. We have seen above that the minimum multipolarity for given values of $I_l$ and $I_f$ is $L = 1$, $I_l - I_f$. If $\Pi_l$ and $\Pi_f$ denote the parities of the initial and final states, then the transitions are classified as

- Parity-favoured if $\Pi_l \Pi_f = (-1)^{I_l - I_f}$
- Parity-unfavoured if $\Pi_l \Pi_f = (-1)^{I_l - I_f + 1}$

In the case of parity-favoured transition for $I_l \neq I_f$, the electric radiation predominates with $L = |I_l - I_f|$ over the magnetic radiation with $L = |I_l - I_f| + 1$ except when $I_l$ or $I_f = 0$.

For parity-unfavoured transition, the magnetic radiation generally predominates with $L = |I_l - I_f| + 1$ over the electric radiation with $L = |I_l - I_f| + 1$ except when $I_l$ or $I_f = 0$.

The case of $I_l = I_f$ is an exception to the rule. In this case $\Pi_l = -\Pi_f$ transitions give pure electric dipole radiation while $\Pi_l = \Pi_f$ generally gives pure magnetic dipole radiation.

The $\gamma$-rays emitted by nuclei have usually energies of the order of some tens of keV to several MeV. Their wavelengths are

$$
\lambda = \frac{c}{v} = \frac{3 \times 10^8 \times 6.626 \times 10^{-34}}{1.6 \times 10^{-13}} E(Hz) = 12.42 \times 10^{-13} \frac{E(MeV)}{E(MeV)} \text{metres}
$$

This means that $\lambda = \gamma \times 2\pi$ varies from about $1.2 \times 10^{-10}$ m to $1.2 \times 10^{-14}$ m for $E_{\gamma} = 0.01$ MeV to 10 MeV. The nuclear radius $R \cdot 10^{-14}$ m is then small compared to $\lambda$ and hence the ratio $R/\lambda << 1$. Since the linear dimensions of the charge distribution in the nuclei are $R \cdot R$, the intensities of the successive orders of multipole transitions decrease by a factor of the order of $(R/\lambda)^2 \cdot 10^{-3}$ for $E_{\gamma} > 0.5$ MeV. Thus the higher order multipole radiations are much weaker and the lowest order multipole transitions usually predominate.

The probability of radiative transition for multipoles of different orders have been calculated by Blatt and Weisskopf.*

The transition probability for the multipolarity $L$ is given by (for EL transition) when summed over different $M_j$ and taking the average over $M_j$,

$$
T_{\gamma}(L) = \frac{1}{\tau_{\gamma}} = \left(\frac{\mu_0 c^2}{4 \pi R}\right) \cdot \frac{8 \pi (L + 1)}{L(2L + 1) !!} \left(\frac{\omega}{c}\right)^{2L + 1} B_{\gamma L}(L) \quad \text{(6.9-15)}
$$

where $B_{\gamma L}(L)$ is the reduced transition probability which contains all relevant nuclear factors depending on the characteristics of the initial ($i$) and final ($f$) states. The double factorial $(2L + 1)!! = 1, 3, 5, 7...(2L + 1)$.

$\mu_0 = \frac{4\pi}{10^7}$ H/m is the magnetic permeability of empty space. Similar expression is obtained for ML transition probability. $B_{\gamma L}(L)$ is of course different.

From detailed calculation of the reduced transition probability, it follows that for EL transitions

$$
\frac{1}{\tau_{\gamma}} = \left(\frac{R}{\lambda}\right)^{2L}
$$

(6.9-16)

For ML transitions

$$
\frac{1}{\tau_{\gamma}} = \left(\frac{R}{\lambda}\right)^{2L + 1}
$$

(6.9-17)

The most allowed is the electric dipole (E1) transition. The next allowed are the electric quadrupole (E2) and magnetic dipole (M1) transitions.

* See Theoretical Nuclear Physics by J.M. Blatt and V.F. Weiskopf. Also see W.E. Burcham, Elements of Nuclear Physics, Ch. IX.
In Fig. 6.13 is shown the multipolarities of radiative transitions in a few typical cases.

![Diagram of radiative transitions]

**Fig. 6.13. Radiative transitions of different types.**

In Fig. 6.13a, the spin of the lowest level is 0 and hence only one value of $L$ is possible for the radiation emitted from any other state. In the transition $2^+$ to $0^+$, the change $\Delta I$ is 2 units with no change of parity. Hence only E2 transition is possible. In the transition $4^+$ to $2^+$, the minimum change in $I$ is 2 units with no change of parity. So the transition is again E2. Other possible values of $L$ are 3, 4, 5 and 6. So the other possible transitions are: M3, E4, M5 and E6 which have much lower intensities and hence are not shown. Similarly the direct transition from the upper most level $4^+$ to the lowest level $0^+$ involves emission of E4 radiation which is far less probable.

In Fig. 6.13b, for the transition $2^+$ to $1^-$, $\Delta I = 1, 2$ or 3 with change of parity. So it is an E1 (electric dipole) transition. Other possible transitions are M2 and E3 which are highly suppressed. The transition $1^+$ to $0^+$ is obviously E1.

In Fig. 6.13c, for the transition $2^+$ to $1^+$, $\Delta I = 1, 2$ or 3 with no change of parity. So this is a mixture of M1 and E2 which have comparable probabilities of transition. Higher order multipole transition M3 is highly suppressed. The transition $1^+$ to $0^+$ is obviously M1.

Finally in Fig. 6.13d, $\frac{1}{2}^+$ to $\frac{3}{2}^+$ with $\Delta I = 1$, yes, is predominantly E1. The other possibility M2 with $\Delta I = 2$ is highly suppressed. The transition $1/2^+$ to $1/2^+$ with $\Delta I = 1$, yes, can be E1 only. Again the transition $\frac{3}{2}^+$ to $\frac{1}{2}^+$ with $\Delta I = 1$, no, is either M1 or E2 with equal probabilities.

![Graph of half-life vs gamma-ray energy]

**Fig. 6.14. Dependence of the half-life of radiative transition on $\gamma$-energy for $A = 100$.**

Gamma Rays

In Fig. 6.14 is shown the plots of $\gamma$-half-lives according to single particle Weisskopf formula given above for the mass number $A = 100$ for different multipolarities, both electric and magnetic, which bear out the results broadly summarized in the above paragraphs.

6.10 Nuclear isomerism

From Fig. 6.14 we see that for high multipolarity (large $L$) and low energy ($E$), the radiative transition half-lives are long and hence the probabilities of such transition low. Such long lived excited nuclear states are known as isomers. The values of $Z$ and $A$ remain unchanged after isomeric transition. In many cases isomeric transitions are associated with the emission of internal conversion electrons (see § 6.11). There is no essential difference between isomeric states and other excited nuclear states undergoing radiative transitions. Only the isomeric states have much longer life-times ($\tau > 10^{-7}$ s) compared to the latter.

![Graph of isomeric transitions]

**Fig. 6.15. Isomeric transitions in some nuclei.**

A few typical examples of isomeric transition are shown in Fig. 6.15.

An example of nuclear isomerism shown in Fig. 6.15a is that of $^{80}$Br isotope formed by the artificial transmutation of the stable isotope $^{79}$Br ($Z = 35$) by slow neutron capture. Two different half-lives of $^{80}$Br are
known viz. 17.6 min and 4.4 hr. The state of half-life 4.4 hr is the isomeric state written as $^{80}\text{Br}^m$. The energy level diagram and the transition is shown in Fig. 6.15 a. The isomeric state goes down to the lower state of half-life 17.6 min by two successive $\gamma$-emissions through an intermediate state of half-life 7.3 nanoseconds ($10^{-9}$ s). The lower state $^{80}\text{Br}$ undergoes $\beta^-$ decay to $^{80}\text{Se}$. In some cases it undergoes $\beta^+$ decay to $^{80}\text{Kr}$. The lower state of $^{80}\text{Br}$ is genetically related to the isomeric state $^{80}\text{Br}^m$ of half-life 4.4 hr. The angular momentum difference between the isomeric state ($5^+$) and $^{80}\text{Br}$ state ($1^+$) shows that it is an M4 transition and hence is highly suppressed, the energy difference being also very small.

In Fig. 6.15 b is shown the isomeric transition of $^{116}\text{In}$ ($Z = 49$). The isomeric state $^{116}\text{In}^m$ undergoes transition to different excited states of $^{116}\text{Sn}$ with the half-life 54 min. On the other hand, the ground state $^{116}\text{In}$ undergoes $\beta^-$ decay to the ground state of $^{116}\text{Sn}$ with a half-life of 13s. There is no genetic connection between the two states of $^{116}\text{In}^m$.

Historically, the first case of nuclear isomerism was discovered by Otto Hahn (1921) in the $^{234}\text{Pa}$ ($Z = 91$) isotope (UZ). It is found that the $\beta^-$ active $\text{UX}_1$ ($^{234}\text{Th}$) usually undergoes the following transformation:

$$\text{UX}_1 \xrightarrow{\beta^-} \text{UX}_2 \xrightarrow{\beta^-} \text{U} \rightarrow \text{U} \text{II}$$

However in about 0.12% case, a different kind of radioactive decay of $\text{UX}_1$ is observed :

$$\text{UX}_1 \xrightarrow{\beta^-} \text{UZ} \xrightarrow{\alpha} \text{U} \text{II}$$

Thus the products $\text{UX}_2$ and $\text{UZ}$ of the decay of $\text{UX}_1$ have two different half-lives. Both $\text{UX}_2$ and $\text{UZ}$ have the same values of $Z$ and $A$ so that they are both $^{234}\text{Pa}$ isotope. Only difference is in their radioactive property (half-lives). The energy level diagram and the isomeric transitions in this case are shown in Fig. 6.15 c. It can be seen that the two states of $^{234}\text{Pa}$ undergo $\beta^-$ decay to form the product nucleus U II ($^{234}\text{U}$). The upper state of half-life 1.18 min is the isomeric state $^{234}\text{Pa}^m$ (UX). There is also radiative transition from this state to the lower state $^{234}\text{Pa}$ (UZ).

Isomeric states are formed following the radioactive decay of a parent nucleus, as in the case of $^{234}\text{Pa}^m$. Sometimes they are produced in artificial transmutation of nuclei (nuclear reaction) as in the case of $^{80}\text{Br}^m$ or in the case of $^{116}\text{In}^m$ which is produced by the slow neutron capture of the stable $^{115}\text{In}$ isotope.

Isomeric states are also observed in stable nuclides. An example is the $7/2^+$ state of $^{109}\text{Ag}^m$ of the stable isotope of $^{109}\text{Ag}$ ($Z = 47$) of silver formed by the E.C. transformation of $^{109}\text{Cd}$ ($Z = 48$). The isomeric state goes down to the ground state $1/2^-$ of $^{109}\text{Ag}$ by E3 radiative transition shown in Fig. 6.15 d.

**Gamma Rays**

Occurrence of the isomeric states having large angular momentum difference from its lower levels is closely linked to the nuclear shell structure to be discussed in Ch. IX. The nuclear isomers are found to be preferentially grouped in the regions just below the major closed shells at $Z$ or $N = 50$ and 82 and at $N = 126$. Such groupings are known as islands of isomerism. We shall discuss about this is Ch. IX.

There is another kind of isomerism, known as shape isomerism observed in some heavy nuclei undergoing spontaneous fission.

### 6.11 Internal conversion

While discussing about the $\beta$-ray spectra in Ch. V we saw that in some cases the continuous $\beta$-ray spectra are crossed by one or more sharp peaks, which are due to electron groups of certain definite energies (see Fig. 5.9). These are known as internal conversion electrons or simply conversion electrons.

The emission of such monoenergetic conversion electron groups can be understood as follows. The $\alpha$ or $\beta$ transformation of a nucleus leaves the latter in a definite state of energy $E_1$ (say) which may be either an excited state or the ground state of the nucleus. If it is an excited state, it deexcites within a time of the order $10^{-13}$ s by the emission of electromagnetic radiation ($\gamma$-rays) by transition to a lower state of energy $E_2$ (say). The $\gamma$-ray photon has thus an energy

$$h\nu = E_1 - E_2$$

There is however an alternative process by which the excited nucleus can deexcite itself. The surplus energy ($E_1 - E_2$) of the nucleus may be directly transferred to an electron in one of the atomic shells of the same atom, which may thus be emitted from the atom, if the binding energy of the electron $B_e$ is less than the energy ($E_1 - E_2$) so transferred. The electron emitted is known as the internal conversion electron. Since the K-shell is nearest to the nucleus, the probability distribution of the electron in this shell has a fairly large amplitude at the position of the nucleus ($r = 0$), so that the surplus nuclear energy will go more often to the K-electron than to the electrons in the outer L, M etc. shells. So the K-conversion process has usually the largest probability. The kinetic energy of the emitted conversion electron is obviously

$$E_e = E_1 - E_2 - B_e = h\nu - B_e$$

Here $h\nu$ is the energy of the photon emitted when the transition between the states $E_1$ and $E_2$ of the nucleus is radiative (i.e., accompanied by $\gamma$-emission). If ($B_e)_K$, ($B_e)_L$, etc. are the binding energies of the electrons in the K, L, etc. shells, then the corresponding kinetic energies of the electrons will be

$$(E_e)_K = h\nu - (B_e)_K$$

$$(E_e)_L = h\nu - (B_e)_L$$

$$(E_e)_M = h\nu - (B_e)_M$$

and so on.

We can then write

$$h\nu = (E_e)_K + (B_e)_K = (E_e)_L + (B_e)_L = (E_e)_M + (B_e)_M = \ldots$$
The difference in the energies of the different conversion electron groups for a given nuclear transition is exactly equal to the energies of the characteristic X-rays of the atom from which the conversion electron is emitted. The emission of the K conversion electron leaves a vacancy in the K-shell, which is then filled up by the transition of an L, M etc. electron, resulting in characteristic X-ray emission. As an alternative Auger electrons may also be emitted.

Since \((E_{\gamma})_L < (E_{\gamma})_K\), we have \((E_{\gamma})_L > (E_{\gamma})_K\). The energy difference \((E_{\gamma})_L - (E_{\gamma})_K = (B_e)_L - (B_e)_K\) is equal to the energy of the characteristic K X-ray emitted, as stated above.

Obviously the conversion electrons from a particular atomic shell will be monoenergetic, since \(h\nu = E_1 - E_2\) has the same value for transitions between two definite nuclear states and the binding energy of the electron \(B_e\) for a particular shell has also a definite value.

As we have seen in § 6.7, it is possible to determine the \(\gamma\)-ray energy \(E_1 = E_2\) by measuring the energy of the conversion electrons by means of a magnetic spectrometer.

If \(\Gamma_\gamma\) and \(\Gamma_e\) are the radiative width and the conversion electron width of the level, then its total width is

\[ \Gamma = \Gamma_\gamma + \Gamma_e \]  

(6.11-5)

Here \(\Gamma_\gamma\) and \(\Gamma_e\) are proportional to the probabilities of transition from the excited level by the two alternative processes of \(\gamma\)-emission and conversion electron emission. \(\Gamma\) is proportional to the probability of transition from the level, assuming that no other process of deexcitation of the level takes place. Obviously \(\Gamma = c/\tau\), \(\tau\) being the mean life of the level.

If in a given interval of time, \(N\) nuclei undergo transition from the state \(E_1\) to the state \(E_2\), and out of these \(N_\gamma\) make radiative transitions while \(N_e\) make transitions by the emission of conversion electrons, then \(N = N_\gamma + N_e\). We define the internal conversion coefficient as

\[ \alpha = \frac{N_e}{N_\gamma} = \frac{\Gamma_e}{\Gamma_\gamma} \]  

(6.11-6)

There may be cases in which the transition between the two levels takes place by \(\gamma\)-emission only. In this case \(N_\gamma = 0\) and \(N = N_e\) so that \(\alpha = 0\). On the other hand, there may be cases in which the transition takes place by the emission of the conversion electrons only. In this case \(N_\gamma = 0\) and \(N = N_\gamma\) so that \(\alpha = \infty\). Thus the conversion coefficient may vary from 0 to \(\infty\). Conversion coefficient corresponding to emission of the electrons from the K, L, etc. shells are designated by the symbols \(\alpha_K, \alpha_L, \alpha_M\) etc. Usually \(\alpha_K > \alpha_L > \alpha_M\) etc. which shows that K-conversion is the most probable. Total conversion coefficient is given by

\[ \alpha = \alpha_K + \alpha_L + \alpha_M + \ldots \]

\(\text{Gamma Rays}\)

Measurement of the conversion coefficient gives valuable information about the multipolarity \(L\) of the transition. The value of \(\alpha\) increases with increasing \(L\) and with decreasing energy of transition, as the following approximate expression shows (see Theoretical Nuclear Physics by Blatt and Weisskopf):

\[ \alpha_{\text{K(EL)}} = Z^3 \left( \frac{e^2}{4\pi e_0 h c} \right)^4 \frac{2m_e c^4}{h^2 \omega} \omega^{3/2} \]  

(6.11-7)

The above expression holds for parity-favoured EL transition in the limit of \(Z e^2/4\pi e_0 h c \ll 1\) and transition energy \(h\omega \gg B_e, h\), the K-shell binding energy.

For the magnetic multipole (ML) transition, we get

\[ \alpha_{\text{K(ML)}} = Z^3 \left( \frac{e^2}{4\pi e_0 h c} \right)^4 \frac{2m_e c^4}{h^2 \omega} \omega^{1/2} \]  

(6.11-8)

The conditions favouring K-conversion are very similar to the conditions favouring the occurrence of isomeric states (see § 6.10). Hence, as already mentioned, the isomeric transitions are accompanied by conversion electron emission.

In general, the value of the conversion coefficient is larger for low transition energy, high multipolarity (\(L\)) and for high Z elements. For the same energy and \(L\), the magnetic multipole transition is more probable than the electric multipole transition. In Fig. 6.16 is shown the variation of \(\alpha\) for different electric multipoles.

![Fig. 6.16. Dependence of the internal conversion coefficient on \(E_\gamma\) for different multipoles](image)

Internal conversion may also occur from the L-shell of the atom. L-conversion is in general less probable than the K-conversion, since the probability of an L-electron being close to the nucleus is much less than that of a K-electron. The ratio of K to L conversion coefficient decreases with increasing multipole order \(L\). For the same \(L\), the ratio is larger for the magnetic radiation than for the electric radiation.
It may be noted that the term internal conversion is a misnomer. It was thought in the early days that in this process, a γ-ray of energy $\hbar \nu = E_1 - E_2$ is at first emitted from the nucleus, which is then absorbed by an orbital electron in the same atom; the latter is then emitted by photoelectric process. Later it was realized that such a conversion process would be extremely rare, since its probability would be equal to the product of the probabilities of the two processes, viz. γ emission probability and photoelectric absorption probability of the γ-photon. The picture we have at present is that no γ-ray is at all emitted in the internal conversion process; the surplus energy of the nucleus is directly transferred to an orbital electron.

6.12 Internal pair creation

If the energy of transition $(E_1 - E_2)$ exceeds $2m_e^2 c^2 = 1.02$ MeV, then sometimes a pair of electron and positron is produced, instead of a radiative transition between the two states. This is known as internal pair creation and is an alternative to the radiative transition. Since the pair is created in the electric field of the nucleus, it is independent of the electron-probability density at the nucleus, i.e., independent of $Z$. The probability of internal pair creation increases with increasing transition energy and is larger for smaller multipolarities. These conditions are thus just opposite to the conditions which favour internal conversion.

Internal pair creation must be distinguished from the pair creation by the γ-ray (when radiative transition occurs) of energy $E_1 - E_2 > 2m_e^2 c^2$ in some other nucleus. Since the latter process is more probable for a high $Z$ medium (see § 6.5), internal pair creation is easiest to observe in light nuclei.

6.13 Zero-zero transition

We saw in § 6.9 that the radiative transition $I_1 = 0$ to $I_2 = 0$ (electric dipole, E0) is strictly forbidden. However, if the orbital electron-probability distribution has an appreciable amplitude at the nucleus, as in the case of the K-electrons, such a transition may take place with the emission of the conversion electron only. Since no radiative transition occurs, the conversion coefficient $\alpha_K = \infty$ so that there is 100% conversion. A conversion electron peak of energy $E_1 - E_2 - (B_K)$ is observed in this case. The lifetime of the excited level is usually quite long. Such transitions have been observed in heavy nuclei for low transition energies. An example is the 0.72 MeV $0^+$ state in $^{72}$Ge decaying to the $0^+$ ground state.

In light nuclei, for energies exceeding $2m_e^2 c^2$ internal pair creation may take place more readily in $0 \rightarrow 0$ transition. Thus the 6.06 MeV $2^+$ state of $^{190}$O nucleus decays to the $0^+$ ground state in this way.

Notice that in the above transitions, no change of parity is involved. No case is known in $0 \rightarrow 0$ transition with parity change such as a $0^+ \rightarrow 0^-$ transition.

6.14 Coulomb excitation

Gamma rays are emitted when an excited nucleus makes transitions to lower energy states. Higher energy states in the nucleus may be excited following alpha or beta decay. They may also be excited when a nuclear projectile enters into the nucleus. There may be an alternative process in which a charged particle passing by a nucleus may excite the latter to a higher energy state without actually entering the nucleus. This is known as the Coulomb excitation. Coulomb excitation may be regarded as an inverse of the process of internal conversion discussed earlier with the important difference that in the latter an electron is emitted in the non-radiative transition while Coulomb excitation is caused by the passage of a heavier ion.

If the energy of the charged projectile is so low that it has negligible probability of entering into the nucleus, then the process is easily distinguished from nuclear effects. At such low energies, the motion of the incident particle of charge Ze can be treated classically. The trajectory of the particle repelled by the inverse-square type of force $Ze^2 / 4 \pi \varepsilon_0 r^2$ due to the target nucleus of charge Ze is a hyperbola determined by Rutherford's theory of α-particle scattering. Of course the projectile is inelastically scattered in this case, since it loses some energy to produce the excited state of the target nucleus. However, as long as the energy-loss is small compared to its kinetic energy, the trajectory will still be close to the same hyperbola.

The electric field at the nucleus due to the incident charged projectile varies with time. The characteristic time for which the interaction becomes effective is $T = a / \nu$ where $a$ is the least distance of approach of the incoming particle for head-on collision and $\nu$ is its velocity. For the transition to take place, this time must be short compared to the time for the averaging of the fluctuations in the orientations of the electric quadrupole moment of the deformed nuclei in which the Coulomb excitation is mostly observed. This latter time is of the order of $\tau = \hbar / \Delta E $ where $\Delta E = E$ is the energy involved in the transition. $\nu$ is the characteristic frequency of the fluctuations in the random orientations. Thus the condition determining the Coulomb excitation to take place is $T << \tau$.

The cross-section for the Coulomb excitation is determined by the trajectory of the incident particle and on the radiation-width of the level. Larger the level-width, greater is the probability of Coulomb excitation.

The cross-section for EL type transition comes out to be

$$\sigma(EL) = \left( \frac{Ze}{4 \pi \varepsilon_0 \hbar \nu} \right)^2 a^{-2l + 2} B_d(EL) f(E, Z, \Delta E) \quad (6.14-1)$$

where $E$ is the energy of the incident particle, and $\Delta E = E_i$ is excitation energy of the level. $f(E, Z, \Delta E)$ is a function which has been evaluated and are available in tabular forms (see K. Alder, A. Bhor, T. Huus, B.
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Mottelson and A. Winther, Rev. Mod. Phys. 30, 1950, pp. 353) \( B_{1e} (EL) \) is the reduced transition probability (see Eq. 6.9-14). The cross section for the magnetic multipole transition is inhibited by a factor \((\omega/c)^2\) for the same multipole order \(L\).

Coulomb excitation has been observed for a large number of nuclei from low to high Z. Bombarding particles include \(\alpha\)-particles and other heavier ions, e.g., \(^{14}\)N and \(^{40}\)A. Many Coulomb excitation processes involving E2 type transitions have been observed, which thus permit the study of the low lying rotational levels in deformed nuclei (see Ch. IX). Such nuclei have internal quadrupole moment \(Q_0\) which can thus be measured by a study of the Coulomb excitation cross-section, since \(B(E2)\) in this case depends on \(Q_0^2\). In this way \(Q_0\) for \(^{114}\)Cd has been determined to be 0.5 barn.

The properties of nuclear states formed can be determined by observations on the inelastically scattered projectiles, intensity and angular distribution of the \(\gamma\)-rays emitted etc. Experiments give the values of energy, spin and parity of the nuclear state involved, as also the transition probability \(B_{1e}(EL)\).

6.15 Inelastic scattering of electrons

In § 2.5, we have seen that experiments on the scattering of high energy electrons yield valuable information on the nuclear charge distribution and nuclear radius. Similar experiments involving inelastic scattering of electrons can throw considerable light on the structure of excited nuclear states.

Eqs. (2.5-1) and (2.5-2) give the elastic scattering cross-sections in terms of the Mott scattering cross-section \(\sigma_x 0\) : \(\sigma(\theta) = \sigma_x 0 [F(q)]^2\).

Here \(F(q)\) is the form factor. A similar expression can be written for the inelastic scattering cross-section, involving the inelastic form-factor \(F_{in}(q)\) where \(q\) is determined by the momentum transfer in the collision:

\[ q = (p - p')/\hbar \]

The form-factor is determined by the Fourier transform of the transition density which depends on the wave functions of the initial (\(i \equiv 0\)) and final (\(f\)) states of the target nucleus:

\[ \rho_f(r) = \psi_f^*(r) \psi_i(r) \]

...(6.15-1)

We can then write

\[ F_{in}(q) = \frac{1}{Z^e} \iint \rho_i(r) \exp(izr) d^3r \]

...(6.15-2)

\(\rho_i(r)\) can be found by calculating the inverse transform of the form factor determined experimentally at different electron energies. These can then be compared with the theoretical values calculated using Eq. (6.15-1) if the wave functions \(\psi_i\) and \(\psi_f\) are known. According to the single particle model, these wave functions correspond to the initial and final states of the nucleon undergoing transition.

6.16 Resonance fluorescence

Fluorescence is the phenomenon of the emission of longer wavelength radiation induced by radiation of shorter wavelength. When a radiating system like an atom or a nucleus undergoes radiative transition from an initially excited state 2 to a final state 1, the energy-difference \(E_0 = E_2 - E_1\) between the two states is distributed between the emitted photon and the recoiling emitter.

\[ E_0 = E_\gamma + E_r \]

Thus the energy of the emitted photon \(E_\gamma = E_0 - E_r\) is slightly lower than the transition energy \(E_0\). The difference is usually small, being almost negligible for the atomic case. We can estimate this difference \(E_r\) from the following simple considerations.

The emitted photon and the recoiling emitter carry equal and opposite momenta (see Fig. 6.17). So that \(p_\gamma = -p_r\). Hence the recoil energy \(E_r\) of the emitting atom (or nucleus) is

\[ E_r = \frac{p_r^2}{2M} = \frac{E_0^2}{2M} - \frac{E_2^2}{2Mc^2} \]

...(6.16-1)

Here \(M\) is the mass of the atom and \(p_\gamma = E_\gamma/c = h\nu/c\).

In resonance fluorescence, the radiation emitted by an atom is absorbed by another atom to induce transition in the latter between the same two states, i.e., from 2 to 1, as shown in Fig. 6.18. Obviously this is possible only if the emitted photon in the first atom has the same energy as the energy absorbed for the transition to take place in the second atom. But as seen above, the energy of the emitted photon \(h\nu\) is slightly less than the transition energy \(E_0\) by \(\Delta E = E_r\). Actually because of the recoil of the absorbing atom, the deficiency in energy due to the recoil effect is twice the above estimate, amounting to \(\Delta E = 2E_r = E_0^2/Mc^2\) (see Fig. 6.18 b).
Now the energy levels of the atoms and nuclei have intrinsic widths \( \Gamma \), determined by the uncertainty relation: \( \Gamma = \hbar / \tau \). Here \( \tau \) is the mean life time of the state. Obviously the ground state has no width since \( \tau = \infty \). All other states are broad as shown in Fig. 6.19. If this width is sufficiently large compared to the deficiency in the energy \( \Delta E \) of the photon from the transition energy \( E_0 \), it may still be possible for the transition to take place. This is the case for the atomic transitions as the following numerical estimates show (see Fig. 6.18a).

\[
\Delta E = 2E_r = \frac{E_v^2}{Mc^2} = 0.21 \text{ eV}
\]

If the life of the state is \( \tau = 10^{-10} \text{ s} \), then the intrinsic width is

\[
\Gamma = \frac{\hbar}{\tau} = 10^{-5} \text{ eV}
\]

Thus \( \Gamma \ll \Delta E \) in the case of nuclei, as shown in Fig. 6.19 b. Thus, if Doppler broadening is neglected, the deficiency \( \Delta E \) in the energy of the emitted photon from the transition energy \( E_0 \) is so large that even with the intrinsic width of the level being taken into account, the photon will be unable to induce the transition in a second nucleus. Thus resonance fluorescence should be a rare phenomenon for nuclear transition. This is confirmed experimentally. However, there are some methods of inducing resonance fluorescence which are discussed below.

(a) Doppler broadening of the levels helps in inducing resonance fluorescence. The estimated Doppler broadening due to thermal motion in the case considered above for \( T = 300 \text{ K} \) is \( D_T = 0.1 \text{ eV} \) which is of the same order of magnitude as \( \Delta E \). Since both the source and absorber nuclei take part in thermal motion, the effect of Doppler broadening is double the above estimate. So the recoil energy-loss can be compensated by heating the source, which increases the Doppler broadening.

The relative magnitudes of the temperature dependent Doppler-width and recoil-shift for the atomic and nuclear levels are compared in Fig. 6.20.

\[
\frac{\hbar}{c} = E_r \frac{v}{c} = \frac{E_v^2}{Mc^2}
\]

(b) Another method of compensating for the recoil energy loss is to move the source of radiation with very high velocity, which can be done by mounting the source on the tip of a high speed rotor. Due to the motion of the source, the frequency of the radiation affects Doppler shift. For motion towards the detector, the frequency is increased by \( v_{rel} / c \) where \( v_{rel} \) is the original frequency and \( v \) is the velocity of the source. As a result, the frequency of the radiation increases. To compensate for the recoil energy loss, the required velocity of the source is determined by the relation

\[
\frac{h \nu}{c} = E_r \frac{v}{c} = \frac{E_v^2}{Mc^2}
\]
which gives

\[ v = \frac{E_\gamma}{Mc} \]

For \( E_\gamma = 100 \text{ keV} \) and \( A = 50 \), we get

\[ v = \frac{1.6 \times 10^{-14}}{50 \times 1.66 \times 10^{-27} \times 3 \times 10^8} = 6.4 \times 10^2 \text{ m/s} \]

(c) A third method is to use a source of radiation which is produced in a preceding nuclear transition (as in \( \alpha \) or \( \beta \) decay) or in a nuclear reaction, which may supply the required velocity to the source to compensate for the recoil energy loss.

Nuclear resonance fluorescence was first observed by P.B. Moon and his collaborators at Birmingham in England in 1951.

The source which was radioactive \(^{198}\text{Au} \) (\( Z = 79 \)) isotope, was mounted on the tip of a high speed rotor of 15 cm diameter and the resonance was looked for in the radiation scattered from a \( \text{Hg} \) (\( Z = 80 \)) absorber. The intensity of the scattered radiation was found to be maximum at a rotor speed of 700 m/s at the periphery. At this speed, the recoil energy loss is compensated which amounted to about 0.8 eV. A width of \( \Gamma = 2.1 \times 10^{-3} \text{ eV} \) for the 0.412 MeV \(^{198}\text{Hg} \) level was derived. This corresponds to a mean life \( \tau = \frac{\hbar}{\Gamma} \) or \( 3.1 \times 10^{-11} \text{ s} \) for the level.

The heating method was used by F. Metzger and W.B. Todd (1953) to measure the width of the 161 keV level in \(^{117}\text{Sn} \). The source was heated in an oven from 270°C to 700°C, which increased the scattering by 1%. The mean life of the state could be determined with an accuracy of ~ 40%.

The third method of compensation by using reaction kinetics was achieved by Metzger and his group for many nuclei. For example, the 1.61 MeV level in \(^{25}\text{Mg} \) (which is a \( 7/2^+ \) spin level in the \( K = 5/2 \) rotational band) was excited by them by inelastic proton scattering (1961) \(^{25}\text{Mg}(p, p')^{25}\text{Mg} \). The deficiency in the emitted \( \gamma \)-ray energy is compensated due to Doppler shift resulting from the motion of recoil of the source (\(^{25}\text{Mg} \)). The method is fairly accurate. In the case cited, a mean life of \( 2.5 \times 10^{-14} \text{ s} \) was measured for the excited level.

Fig. 6.20a shows that the overlapping between the emission and absorption curves for the atomic case is almost complete. On the other hand, as is seen in Fig. 6.20b for the nuclear case, the region of overlapping between the emission and absorption curves is generally small. The region of overlapping can be increased considerably by heating the source, which increases Doppler broadening.

6.17. Mössbauer effect

In 1958, R.L. Mössbauer in Germany, made a very important discovery in the field of gamma-ray physics, which won him the Nobel prize in Physics in 1961.

Mössbauer observed that under certain circumstances, \( \gamma \)-rays could be emitted from nuclei without any loss of energy due to the recoil of the emitting nucleus. As such these \( \gamma \)-rays have the same energy as the transition energy between the two states. This type of transition is known as the recoiless transition and the effect is known as Mössbauer effect.

Mössbauer carried out his experiments using the 129 keV \( \gamma \)-rays from \(^{191}\text{Ir} \) (\( Z = 77 \)) produced in the \( \beta^- \) decay of \(^{190}\text{Os} \) (\( Z = 76 \)). The energy level diagram for \(^{191}\text{Ir} \) is shown in Fig. 6.21a. The 129 keV excited level in \(^{191}\text{Ir} \) has a mean life of \( 1.3 \times 10^{-10} \text{ s} \) so that it has a width \( \Gamma = \hbar/\tau = 5 \times 10^{-6} \text{ eV} \). Since the transition is to be ground state of \(^{191}\text{Ir} \) (stable) the natural width of the 129 keV \( \gamma \) line is also the same. In a recoiless transition, the emitted \( \gamma \)-ray is highly monochromatic and hence can induce resonance transition in another identical nucleus, provided the absorption process is also recoiless.

Mössbauer could produce recoiless transitions by embedding the emitting atoms in a solid lattice. The recoil energy is low enough (~ 0.05 eV) so that the emitting atoms are not dislodged from their lattice sites. In this case, the recoil is taken up by the target atom, but by the entire crystal. As the mass \( M \) appears in the denominator in the recoil energy loss formula (Eq. 6.16-1), the recoil energy loss becomes negligibly small, since \( M \) now corresponds to the mass of the whole lattice and is enormously large compared to the nuclear mass.

Mössbauer's experimental arrangement is shown in Fig. 6.22. The source and the absorber (an iridium foil) were placed in two cryostats maintained at the temperature of 88 K. The cooling reduced the thermal broadening so much that there was no overlapping of the emission intensity and absorption cross section curves (see Fig. 6.19 b) and no resonance absorption would normally be expected. However Mössbauer observed enhanced absorption for reasons explained above.

There was arrangement for rotating the cryostat containing the source. When rotated in one
direction, the source moves towards the absorber while for rotation in the opposite direction, the source moves away from the absorber. Because of Doppler effect, the frequency of the emitted radiation changes when the source moves, which destroys the resonance condition because of the very high degree of monochromaticity of the radiation. The velocity of the source at which the Doppler shift becomes equal to the natural width $\Gamma$ of the level can easily be estimated.

$$D = E_\gamma \frac{\nu}{c} = \Gamma$$

or, $$\frac{\nu}{c} = \frac{\Gamma}{E_\gamma}$$

...(6.17-1)

For the case under consideration, since $\Gamma = 5 \times 10^{-6}$ eV (see above), we get

$$\nu = 5 \times 10^{-6} \quad \frac{c}{129 \times 10} = 4 \times 10^{-11}$$

or,

$$\nu = 0.012 \text{ m/s}$$

The results obtained by Mössbauer showing the variation of the intensity of the γ-ray beam transmitted by the absorber, against the relative velocity of the source and the absorber, is shown in Fig. 6.23. The figure shows that at velocities of a few cm/s, for which the Doppler shift is of the order of $10^{-5}$ eV or less, the resonance condition is disrupted. This confirms that a recoilless transition giving rise to a γ-ray line of intrinsic width $\sim 5 \times 10^{-6}$ eV must have been produced in this experiment.

The resonance absorption method in recoilless transitions can be used in measuring extremely small energy variations. The measure of the accuracy is $\Gamma/E$ which is of the order of $10^{-11}$ or less. Actually the relative accuracy of measurements is even smaller, because it is possible to detect change in absorption for a frequency shift $\sim 1/100$ of the intrinsic line width.

Since the pioneering experiment of Mössbauer, the effect has been observed in many other materials. The most extensively used substance is the iron isotope $^{57}\text{Fe}$ produced in the K-capture decay of $^{57}\text{Co}$ ($\tau = 270 \text{ d}$), for which no cooling is required. The energy level diagram of $^{57}\text{Fe}$ is shown in Fig. 6.21b. The transition from the 14.4 keV level in $^{57}\text{Fe}$ to its ground state produces a γ-ray of energy 14.4 keV which is the most suitable for use in Mössbauer type experiments. The excited level has a relatively long mean life of $1.0 \times 10^{-7}$ s corresponding to an intrinsic width of $6.6 \times 10^{-9}$ eV of the γ-ray line. The absorber is usually an iron sheet enriched in the $^{57}\text{Fe}$ isotope (stable). This increases the probability of recoilless transition. In this case the resonance condition is destroyed at a source velocity of only 0.14 mm/s.

Another substance frequently used is the $^{119}\text{Sn}$ isotope of tin for which $E_\gamma = 23.8 \text{ keV}$, $\tau = 2.6 \times 10^{-8}$ s and $\Gamma = 2.5 \times 10^{-8}$ eV so that $\Gamma/E_\gamma = 10^{-12}$.

The conditions favouring recoilless transitions are lower transition energy, lower crystal temperature and high Debye temperature.

**Theoretical considerations:**

The explanation of Mössbauer effect is based on the quantum theory of the specific heat of solids, first proposed by Einstein and later modified by P. Debye (see Vol. 1). Einstein's theory assumes that a solid consisting of $N$ atoms is a set of $3N$ harmonic oscillators all of the same frequency $\nu = \omega/2\pi$ where $\omega$ is the circular frequency. $\omega$ is determined by the condition $\hbar \omega = kT_\nu$ where $k$ is the Boltzmann constant and $T_\nu$ is the Einstein temperature which is a characteristic constant of the solid, determined by its atomic mass and the elastic constants.

If the solid consists of atoms with excited nuclei, then the emission of a γ-ray by such a nucleus will lead to the recoil of the latter. The recoil energy, in general, is different from that in the case of emission from a nucleus in a free atom. However, calculations show that the mean recoil energy, even in this case is equal to recoil energy of a free nucleus $<E_r> = E_\gamma^2/2Mc^2$ where $M$ is the nuclear mass. If $<E_r> < \hbar \omega$ then some of the γ-transitions will be recoilless (no excitation of the oscillator) since the minimum energy transfer into the oscillator is $\hbar \omega$ so that in some cases the photon emission will not lead to any energy transfer to the emitting nucleus which will thus fail to excite the oscillators.

We can express the condition $<E_r> < \hbar \omega$ by writing

$$<E_r> = (1 - f) \frac{\hbar \omega}{10^{-12}}$$

...(6.17-2)

so that $f = 1 - <E_r>/\hbar \omega$, which is known as the Mössbauer coefficient and gives the fraction of recoilless transitions.

Since the total energy of an oscillator is twice the mean kinetic energy $E_k$, and since at $T = 0 \text{ K}$, the zero point energy is $E_{osc} = \frac{\hbar \omega}{2}$, we can write

$$\hbar \omega = 2E_{osc} = 2 <E_k> = \frac{\rho^2}{2M}$$

...(6.17-3)

where $\rho$ is the momentum. From Eq. (6.16-1), we have
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\[
\langle E_r \rangle = \frac{p^2}{2M} = \frac{\hbar^2}{2M} \frac{\langle x^2 \rangle}{\hbar^2} = \frac{\langle x^2 \rangle}{\hbar^2} \quad \text{(6.17-4)}
\]

where \( \hbar = \lambda / 2\pi \), \( \lambda \) being the wavelength of the radiation. From Eqs. (6.17-3) and (6.17-4) we then have

\[
\frac{\langle E_r \rangle}{\hbar \omega} = \frac{\hbar^2}{\hbar^2} \frac{\langle x^2 \rangle}{\hbar^2} = \frac{\langle x^2 \rangle}{\hbar^2} \quad \text{(6.17-5)}
\]

where \( \langle x^2 \rangle \) gives the mean squared position of the oscillator. Here we have made use of the uncertainty relation \( \Delta x \cdot \Delta p = \hbar / 2 \). For the oscillator

\[
\Delta x = \langle x^2 \rangle^{1/2} - \langle x^2 \rangle = \langle x^2 \rangle \quad \text{and} \quad \Delta p = \langle p^2 \rangle - \langle p^2 \rangle = \langle p^2 \rangle
\]

So we finally get the Mössbauer coefficient as

\[
f = 1 - \frac{\langle E_r \rangle}{\hbar \omega} = \frac{\langle x^2 \rangle}{\hbar^2} \quad \text{(6.17-6)}
\]

A more rigorous treatment gives for the general case (\( T \neq 0 \))

\[
f = \exp \left( -\frac{\langle x^2 \rangle}{\hbar^2} \right) \quad \text{(6.17-7)}
\]

In the limit of \( T \to 0 \), the above expression reduces to the approximate expression (Eq. 6.17-6).

Eq. (6.17-6) shows that the recoilless transition becomes more probable if

(a) Temperature \( T \) is low (for which \( \Delta x^2 \) and hence \( \langle x^2 \rangle \) are quite small);

(b) The emitter is a solid (for which the amplitude of oscillation and hence \( \langle x^2 \rangle \) is small.

(c) Transition energy is low, which makes \( \lambda \) large;

(d) Elastic constants of the material are large, which make the frequency \( \omega \) large so that \( T_0 \) is correspondingly large.

These conclusions are in agreement with observations.

In the more rigorous theory of specific heat proposed by Debye, the oscillators in the solid oscillate with a whole range of frequencies up to a maximum \( \omega_m \) given by \( \omega_m = kT_D \) where \( T_D \) is the characteristic Debye temperature of the solid determined by elastic constants of the crystal.

The theory is now modified with \( T_D \) replacing \( T_E \). For a monatomic crystal, the following expression for \( f \) is obtained:

\[
f = \exp \left[ -6 \frac{\langle E_r \rangle}{kT_D} \left( \frac{T}{T_D} \right)^2 \int_0^{T_D} y \exp y - 1 \right] \quad \text{(6.17-8)}
\]

In the limit of \( T \to 0 \), this reduces to

\[
f = \exp \left( -\frac{3}{2} \frac{\langle E_r \rangle}{kT_D} \right) \rightarrow 1 - 3 \frac{\langle E_r \rangle}{2 \pi \omega_m} \quad \text{(6.17-9)}
\]

\section*{Gamma Rays}

\subsection*{6.18 Applications of Mössbauer effect}

The accuracy of measurement of energy variations attainable by utilizing Mössbauer effect is unbelievably high. As stated before, the accuracy \( \Gamma / E \) is of the order of \( 10^{-11} \) or better. This makes possible the measurement of very small variations in the radiative transition energies in nuclei due to the magnetic fields produced by the atomic electrons or due to inhomogeneous electric fields (hyperfine splitting), effects of charges in the extra nuclear electronic structure in the emitting and absorbing atoms, etc.

\textbf{(a) Hyperfine splitting of the nuclear energy levels:}

Hyperfine splitting of the atomic energy levels arises from the magnetic interactions of the nuclear magnetic moment with the magnetic field due to the orbital electrons at the nucleus (see Ch. VIII). Nuclear magnetic moments are of the order of nuclear magneton \( \mu_N = 5.05 \times 10^{-27} \text{ J/T} \). The average magnetic field at the nucleus due to the electronic current in the atomic shell is

\[
B = \frac{\mu_0 q}{4 \pi r} \times v
\]

Since \( v = l \hbar / mr \) where \( l \) is the azimuthal quantum number, we get

\[
B = \frac{\mu_0 q \hbar l}{4 \pi m r^3} = 2 \text{ tesla}
\]

assuming \( q \) to be the electronic charge and \( l = 1 \). The hyperfine splitting is thus

\[
\Delta E \sim \mu_N \times B = 10^{-7} \text{ to } 10^{-8} \text{ eV}
\]

Since the atomic levels have energies \( \sim 1 \text{ eV} \), the relative magnitude of hyperfine splitting of the atomic energy levels is

\[
\Delta E \over E \sim 10^{-7} \text{ to } 10^{-8}
\]

Thus the hyperfine structures of the atomic spectral lines can be well resolved by optical spectroscopic methods.

On the other hand, the nuclear levels have energies of the order of \( 10^4 \text{ to } 10^5 \text{ eV} \) or more. Hence the value of \( \Delta E / E \) in this case is much smaller:

\[
\Delta E \over E \sim 10^{-11} \text{ to } 10^{-13}
\]

Because of the very small magnitude, such splitting can only be studied with the help of Mössbauer effect.

Hyperfine splitting of a nuclear level was first observed in the case of 14.4 keV \( \gamma \)-ray line of Fe. The two levels \( 3/2^- \) and \( 1/2^- \) which are involved in this transition (see Fig. 6.21 b) split...
up in a magnetic field, as shown in Fig. 6.24, due to space quantization. The transitions between these levels are governed by the selection rules $\Delta m = 0, \pm 1$.

Since the levels in both the emitting and absorbing nuclei undergo hyperfine splitting, the observed Mössbauer pattern would be very complicated. To avoid this, the emitting nuclei are usually mounted on a stainless steel grating which is diamagnetic. Hence there is no hyperfine splitting in them and the observed pattern corresponds to the splitting of the absorber nuclei.

The exciting line from the emitter nuclei is thus a single line, which can produce the six different absorption lines of slightly different frequencies through resonance fluorescence by moving the source with different velocities, which is of the order of 1 mm/s or less (see Fig. 6.25). This type of pattern is observed if the compound Fe$_2$O$_3$ is used as the sample.

Fig. 6.25. Observed hyperfine structure of the nuclear levels in $^{57}$Fe.

The magnetic field at the nucleus of $^{57}$Fe due to the orbital electrons has been found to be $B = 33$ T, assuming $\mu = 0.09 \mu_B$ in the ground state. The magnetic moment of the excited $^{57}$Fe nucleus has also been deduced from this experiment which comes out to be $\mu_{ex} = -0.153 \mu_B$.

Hyperfine splitting of the nuclear energy levels is also produced by the interaction of the nuclear quadrupole moment with the inhomogeneous static electric field produced by the extra nuclear electrons at the nucleus for the nuclear spin $I \geq 1$. This is observed if the compound FeCO$_2$ is used as the sample.

(b) Isomeric or chemical shift:

Mössbauer effect considered so far refers to identical atoms in the emitting and absorbing systems, i.e., the atoms at the lattice sites have identical chemical compositions. Hence whatever effects the electronic environment may have on the nuclear levels are the same in both the emitting and absorbing atoms. If however, the atoms in the two have different chemical compositions, then the nuclear levels will be influenced by different amounts in the emitting and absorbing atoms.

The nuclear energy shift due to the orbital electrons depends on $|\psi(0)|^2$ where $\psi(0)$ is the electronic wave-function at the nucleus. It also depends on the nuclear radii in the ground state and in the excited state ($\Delta E \propto R^2$). Thus, by measuring the chemical shift in the nuclear level

split by the influence of the orbital electrons in the absorber nucleus in relation to the emitting nucleus (both the same) it is possible, in principle, to estimate the nuclear radii $R_e$ and $R_a$ in the ground and excited states of the nucleus. To do this, it is necessary to calculate the values of $|\psi_{ah}(0)|^2$ and $|\psi_{em}(0)|^2$ which can be done under some simplifying assumptions.

Chemical shifts are usually very small ($\Delta E/E \sim 10^{-12}$) and can be determined by Mössbauer method by moving the source at velocities ~ 0.1 mm/s relative to the absorber. It has been found that $R_{ex}$ may be greater or less than $R_e$. For $^{57}$Fe, $R_{ex} < R_e$ by 0–1% while for $^{119}$Sn, $R_{ex} > R_e$ by 0.01%.

(c) Gravitational red shift

In Vol. I, we saw that one of the consequences of Einstein’s General Theory of Relativity is that the frequency of light changes in the gravitational field. Some astronomical experiments have been carried out to test the validity of Einstein’s prediction. Redshift of light coming from the white dwarf companion of the star Sirius has been measured, which seems to be in agreement with the predictions of the General Theory. Since there may be other reasons for the observed red shift, the evidence cannot be regarded as conclusive. It may be mentioned that the white dwarf stars are massive, but have small radii. Hence their gravitational fields are very strong.

R. V. Pound and G.A. Rebka (Jr.) carried out an experiment based on Mössbauer method to measure the effect in the earth’s gravitational field (1960) on the frequency of light. According to the General Theory of Relativity, a photon of energy $E_\gamma = h \nu$ behaves like a particle of mass $m = E_\gamma/c^2$ in a gravitational field. Hence in rising through a height $l$ in the gravitational field of the earth, its potential energy will increase by an amount

$$\Delta E_{gr} = mgl = \frac{E_\gamma}{c^2} g l$$

where $g$ is the acceleration due to gravity. Thus the energy of the photon will decrease by the same amount. So the frequency of the light will decrease by

$$\Delta \nu_{gr} = \Delta E_{gr} / h = E_\gamma g l / c^2 h$$

This decrease causes the light of shorter wavelength to shift towards longer wavelength, an effect known as the gravitational red shift.

In the reverse case, if light moves downwards against the force of gravity, it will suffer a blue shift.

In Pound and Rebka’s experiment, $l = 22.6$ m, which gives,

$$\frac{\Delta E_{gr}}{E_\gamma} = \frac{g l}{c^2} = \frac{9.8 \times 22.6}{(3 \times 10^8)^2} = 2.5 \times 10^{-15}$$

In both cases, the shift is very small, hence it cannot be measured by ordinary methods.
This is an extremely small effect, requiring a relative Doppler velocity of less than 1 micron per second to compensate for the frequency shift. Such values of the velocity were attained by using a hydraulic unit with two pistons of different diameters, the smaller one being moved by a clockwork arrangement.

In later experiments by Pound and Snyder (1965) the accuracy of measurement was improved by using a piezoelectric vibrator to modulate the emitter velocity within the required limits. The results of the measurement are in agreement with the predictions of the General Theory of Relativity.

It may be mentioned that Mössbauer method has been used by Hay, Schiffer, Cranshaw and Egelstaff (1960) to provide experimental verification of time-dilation, predicated by the Special Theory of Relativity.

6.19 Angular correlation experiments

One of the most important methods of determining the spins of the nuclear states is to carry out angular correlation measurements between the gamma rays emitted in cascade.

Consider two nuclear states \( i \) and \( f \) between which transition takes place via an intermediate state, which is very short-lived. The spins of the two states are \( I_i \) and \( I_f \), while that of the intermediate state is \( I \) (see Fig. 6.26 a). Fig. 6.26 b shows the experimental arrangement for measuring the angular correlation between the two gamma rays \( \gamma_1 \) and \( \gamma_2 \) emitted in cascade, as shown in Fig. 6.26 a. S is the source emitting the gamma rays, while C\(_1\) and C\(_2\) are the counters to detect \( \gamma_1 \) and \( \gamma_2 \) respectively by properly gating them so that they can detect only the gamma rays of particular energies \( E_1 \) and \( E_2 \) respectively. The two counters are arranged in coincidence so that a count is recorded only when the output pulses from them enter a suitably designed electronic circuit (mixing circuit) almost simultaneously. For details about coincidence counting arrangement see Ch. VII.

In the experiment, there is a reference direction (z-direction) which is usually the direction of emission SC\(_1\) of the first gamma-ray (\( \gamma_i \)). The second counter C\(_2\) is placed on a rigid arm SC\(_2\) of the apparatus, which can be turned through known angles \( \theta \) w.r.t. to the fixed arm SC\(_1\). The coincidence counting rate \( N(\theta) \) as a function of \( \theta \) is then determined.

\( S_{LM} = \frac{1}{2L(L+1)} \left[ (L(L+1)-M(M+1)) \left| Y_{LM} M+1 \right|^2 + \left| Y_{LM} M-1 \right|^2 + 2M^2 \right] \quad \ldots (6.19.1) \)

The same expression holds for electric and magnetic multipole radiations of order \( L \), \( Y_{LM}^M \) is the spherical harmonic which depends on \( \theta \) and \( \phi \).

If all the sublevels of different \( M \) are equally populated, then the total intensity is isotropic. However, after the first transition, the different \( M \) values are not all equally probable as the simple example above shows. Let us consider the transition from \( I_i = 0 \) to \( I_f = 0 \) via a state of angular momentum \( I = 1 \). Both the transitions 0 to 1 and 1 to 0 are dipole transitions. Hence we have here a dipole-dipole angular correlation. So we have \( L = 1 \) for each. Further \( M = \pm 1 \). The three spherical harmonics involved are

\[ Y_1^1(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} \sin \theta \exp (i\phi) \]

\[ Y_1^{-1}(\theta, \phi) = \sqrt{\frac{3}{8\pi}} \sin \theta \exp (-i\phi) \]

\[ Y_0^0(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos \theta \]

The angular distribution of the coincidence radiation is then given by

\[ W(\theta) = S_{11} + S_{-1} \]

\[ = \frac{1}{4} \left[ \left| Y_1^1 \right|^2 + \left| Y_1^{-1} \right|^2 + \left| Y_0^0 \right|^2 + 2 \left| Y_0^0 \right|^2 \right] \quad \ldots (6.19.2) \]

\[ = \frac{3}{8\pi} (1 + \cos^2 \theta) \]
Similar expressions can be obtained for other values of \( l \).

In the example cited above, it is possible to determine the value of \( l \) unambiguously if the angular correlation is measured.

In the general case of \( I_1 \rightarrow I \rightarrow I_f \) with the multipole orders \( L_i \) and \( L_f \) for the two successive \( \gamma \)-rays, \( W(\theta) \) is given by

\[
W(\theta) = \sum_i A_{2i} P_{2i}(\cos \theta) \quad \ldots (6.19-3)
\]

Here \( P_{2i}(\cos \theta) \) is the Legendre polynomial of order \( 2i \) with \( i = 0, 1, 2, 3 \ldots \). The maximum value \( l_m \) of \( i \) is determined by the minimum of \( I_1, L_i \) and \( L_f \). The coefficients \( A_{2i} \) are given by

\[
A_{2i} = F_{2i}(L_f, I_f, I) F_{2i}(L_i, I_i, I) \quad \ldots (6.19-4)
\]

\( F_{2i} \)'s are certain combinations of the Clebsch Gordan coefficients (see Appendix A (V) and Racah coefficients which have been tabulated by L.C. Biedenharn and M.E Rose in Reviews of Modern Physics, Vol. 25, pp. 729 (1953).

The shape of the angular correlation curves is illustrated for a typical case in Fig. 6.27. As stated, it depends on the multipolarities of the coincidence radiations and hence permits the determination of the spins of the states. It is independent of the parities of the quanta.

**Gamma Rays**

Further, \( \gamma_1 \) and \( \gamma_2 \) must be pure multipole radiations. If this is not true and each of \( \gamma_1 \) and \( \gamma_2 \) is a mixture, then the calculations can be extended to take into account such mixing and the evaluation of the mixing ratio is possible.

Polarization-sensitive detectors permit determination of the parity changes in the transitions.

In the above analysis we have considered \( \gamma-\gamma \) angular correlation. Besides, one can measure \( \alpha-\gamma \) or \( \beta-\gamma \) angular correlations, in which a \( \gamma \)-transition follows immediately after an \( \alpha \) or a \( \beta \)-transition. It is possible to calculate the angular distribution in these cases by considering the probability of emission of the \( \alpha \) or \( \beta \)-ray in the first transition and that of the \( \gamma \)-ray in the second transition.

### 6.20 Measurement of radiative widths

Since the radiative transitions from the excited nuclear states take place in times of the order of \( 10^{-15} \) s, special techniques are required to measure the life-times of these states, from which the \( \gamma \)-ray widths can be deduced using the relation (6.9-1). Some of these methods are discussed below.

(a) **Comparison with \( \alpha \)-decay life-times**: We have seen in § 4.11 that \( \alpha \)-active nuclei, having very short half lives, such as RaC' or ThC' emit long range \( \alpha \)-particles. The \( \alpha \)-decay life-time of ThC' is \( 3 \times 10^{-7} \) s. Comparison of the intensities of the long range \( \alpha \)-particles with the main group (usually \( 1 : 10^5 \)) gives an estimate of the radiative transition half-life of the excited states of ThC' which comes out to be about \( 10^{-13} \) s for the 1.76 MeV state.

(b) **Delayed coincidence method**: This method can be used to measure life-times down to about \( 10^{-11} \) s. It has been discussed in § 3.11 (x).

(c) **Recoil distance method**: This method has also been discussed in § 3.11 (viii) in connection with the measurement of very short \( \alpha \)-decay half-lives.

As an example of its application to the measurement of the life-times of excited nuclear states, we consider the case of \( ^{16}O \) produced in the nuclear reaction \( ^{15}O + p \rightarrow ^{16}O + \alpha \). The product nucleus \( ^{16}O \) recoils and leaves the target with a velocity of \( \sim 10^3 \) m/s. If the target is moved rapidly w.r.t. the corner of a shield protecting a detector till it comes in front of a collimating slit facing the detector, the flight distance before the radiative decay can be determined which gives the life time of the state.

The life time of the 6.06 MeV and 6.13 MeV states of \( ^{16}O \) have been found to be \( 7 \times 10^{-11} \) s and \( 1.4 \times 10^{-11} \) s respectively by this method.

(d) **Doppler shift method**: The method is illustrated in Fig. 6.28.

The projectile beam from the accelerator hits the target to produce the product which undergoes radiative transition. If a stopper is placed close to the target, (position 1), the product nucleus reaches the latter...
Before γ-emission and gets embedded in it. Thus the γ-emission takes place from the product nucleus which is at rest so that the original energy of the transition can be measured by a suitable detector such as a GeLi detector.

![Doppler shift method diagram](image)

**Fig. 6.28.** Doppler shift method of measuring very short life times for radiative transitions.

If however, the stopper is at a fairly large distance from the target (position 2), the γ-ray will be emitted from the product nucleus while it is in flight and hence will undergo Doppler shift by an amount \(\Delta E = (E \cos \theta)/c\) where \(E\) is the transition energy, \(v\) is the velocity of the nucleus and \(\theta\) is the angle between the direction of emission of the γ-ray and the direction of motion of the nucleus. Since \(v/c \approx 10^{-2}\), the value of \(\Delta E\) is several keV for \(E \approx 0.1\) to 0.5 MeV. Measuring the fraction of the unshifted quanta for different positions of the stopper, it is possible to determine the life-time of the transition.

The method is specially suitable for nuclei produced in heavy ion induced reactions (see § 11.16), since higher velocity recoils are produced in these cases, so that shorter life times can be measured.

Doppler shift method can be used to determine life-times in the range \(10^{-12}\) to \(10^{-9}\) s.

(e) In some cases, the resonance fluorescence method discussed in § 6.16 can be used to determine the width of the excited state from which the life-time of the state can be derived.

**Reference**

4. Alpha, Beta and Gamma Ray Spectroscopy, Edited by K. Siegbahn, North Holland Publishing Co. (1965), Chapters XV and XVI; also Chapters XXI A and XXI B.

### Gamma Rays


### Problems

1. Calculate the wavelength and frequencies of γ-rays of energies 0.1, 1.0, 2.6 and 8 MeV.
2. In an experiment on the absorption of γ-rays in lead the half-value thickness is found to be 112 kg/m². What is the mass absorption coefficient of this radiation? If the density of lead be 11.25 x 10³ kg/m³, what is the linear absorption coefficient? (6.19 x 10⁻³ m²/kg; 70.26 m⁻¹)
3. The photoelectrons emitted from platinum by γ-rays from a given source produce a peak at Br = 3.23 x 10⁻⁵ T.m when measured by a β-ray spectrometer. Assuming this to be due to the K-electrons from platinum, find the energy and wavelength of the γ-rays. The K-shell binding energy of platinum may be taken to be 78 keV.
4. Starting from Eq. (6.4-3) show that the maximum energy of the recoil electrons in Compton scattering is given by Eq. (6.4-4).
5. Prove that for Compton scattering at the angles \(\theta_0\) and \(\pi\), the recoil electrons are emitted at the angles \(\pi/2\) and \(\theta_0\) respectively.
6. If the maximum energy of the recoil electrons be 1 MeV in the Compton scattering of a certain γ-ray, what is the energy of the photon? (1.234 MeV).
7. \(^{24}\text{Na}\) decays by β-emission into \(^{24}\text{Mg}\) by the emission of two β-ray groups of maximum energies 1.4 and 4.12 MeV. Two γ-rays of energies 1.38 and 2.72 MeV are also observed following the emission of the lower energy β-group. Draw the possible energy level diagram of \(^{24}\text{Na}\) to explain the above observations.
8. A scintillation counter is calibrated with the help of γ-rays of energies 0.122, 0.511 and 1.278 MeV which produce photopeaks at the channel numbers 100, 350 and 810 respectively. Draw a calibration curve of the γ-energy vs. the channel number of the pulse-height selector and find the energy of an unknown γ-ray which produces a photopeak at the channel number 450.
9. \(^{77}\text{Ge}\) has an isomeric state 1/2⁺ which emits γ-rays of energy 0.38 MeV with a half-life 54 s to go down to the ground state 7/2⁻ which subsequently undergoes β-transformation. What is the multipolarity of the γ-transition? Using Fig. 6.13, show that the half-life is consistent with the multipolarity you assign (Note that you have to correct for the differences in the value of A from that given in the figure) (E3).
10. \(^{18}\text{Hf}\) (Z = 72) undergoes β-transformation to \(^{181}\text{Ta}\) followed by a number of γ-rays which emit K and L conversion electrons of energies 68, 124, 233, 289, 414 and 470 keV. The K and L electron binding energies in Ta are 67.6 and 11.6 keV respectively. Find the γ-ray energies. (135.6, 300.6, 481.6 keV)
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11. Prove that a free electron cannot absorb the entire energy of a photon falling on it.

12. Prove that the electron-position pair creation by a high energy photon cannot take place in vacuum. (Start from the energy and momentum conservation equations).

13. \( ^{77}\text{As} \) has an isomeric state \( 9/2^- \) of energy 475 keV, which undergoes radioactive transition to \( 3/2^- \) ground state and to \( 3/2^- \) excited state of energy 265 keV. Two \( \gamma \)-rays of energies 265 keV and 210 keV are observed. State the nature of the transitions and estimate their relative intensities. (M1, E3)

14. Calculate the recoil energy difference between the emission and absorption lines for the 129 keV level of \( ^{191}\text{Ir} \). What source velocity \( \beta \) required to fully compensate this difference?

15. A nucleus \( X \) undergoes \( \beta \)-transformation to produce an excited nucleus \( Y^* \) which subsequently emits a 200 keV \( \gamma \)-ray. What should be minimum \( \beta \)-energy from \( X \) so that the \( \gamma \)-ray from the recoiling \( Y^* \) may be absorbed by another stationary \( Y \) nucleus?

Detection of Nuclear Radiations and their Measurement

7.1 Introduction

Evolution of the techniques of nuclear radiation detection has played the most vital role in unraveling the mysteries of the atomic nucleus. The radiations coming out of the nucleus, such as the \( \alpha \), \( \beta \) or \( \gamma \)-rays in spontaneous transformations or various types of subatomic particles (both charged and uncharged) in induced transformations are the signals which carry with them information about the properties of the nucleus. Hence their detection and measurement are of prime importance in understanding the structure of the nucleus. The sensitive instruments which have been developed for this purpose over the years thus act as our eyes and ears to probe the mysteries of the nucleus.

Much of the earlier work on radioactive radiations was done by using very simple detecting devices. These have been largely supplanted by more sophisticated devices at present.

The principles of nuclear radiation detection can be broadly subdivided into three classes:

(a) Methods based on the detection of free charge carriers: During the passage of an ionizing radiation through a medium (solid, liquid or gas) both positive and negative ions are produced. Since ionizing radiation comprises charged particles moving with high velocity, the method is primarily applicable in the case of charged particle detection. Uncharged radiation like gamma rays or neutrons can also be detected by instruments based on this method since they usually eject charged particles which then cause ionization in the medium.

Instruments based on this method include ionization chambers, proportional counters, Geiger-Müller counters and semi-conductor detectors.

(b) Methods based on light sensing: These are also applicable for both charged particle detection and detection of uncharged radiation.